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A STATISTICAL ANALYSIS OF CAVE MEANDERS

by

Ⓒ C. CHRISTOPHER SMART

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled A Statistical Analysis of Cave Meanders submitted by C. Christopher Smart in partial fulfilment of the requirements for the degree of

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ABSTRACT

Fluvial meander studies fall into three main areas: form description, process analysis, and regime and hydraulic geometry studies. All these approaches share the problem of handling variables such as discharge and sinuosity, and therefore statistical methods are commonly used.

Some cave passages show similar planform to surface meanders, although bedrock fractures may limit free development. Cave meanders are surveyed in Ireland and New Zealand by direct discretisation of the channel centreline into equal-length segments. Nine bearing series are produced. Width, gradient and transverse erosional marks (scallops) are measured in the Irish caves and fracture location and axis information from New Zealand.

The data are transformed into curvature and change in curvature by differencing. It is shown that statistical tests can not be used to compare the orientation distributions of the passage and fractures. The curvature distribution is possibly more leptokurtic in caves with fracture control. Bearing distributions allow direct calculation of sinuosity from the series mean vector strength. In addition, curvature and change in curvature sinuosity can be calculated to produce a powerful description of planform.

In order to determine meander form, runs of sign of curvature are defined as "individual bends". The method is contingent on a small sampling interval for successful resolution, but generates results loosely compatible with earlier studies. The meander wavelength is found to be inversely related to stream width. Width is dependent on chemical aggressiveness in the cave environment. Sediment may enhance or subdue erosion depending on the former's activity. Boulders protecting the cave floor from solution lead to greater width. Downstream decrease in height and width is a result of chemical saturation. Other aspects of form are considered to result from past changes in hydrology.

Scallop length is interpreted in terms of flow velocity, and acceptable results produced. Scallops cover walls and floors in places, and are assumed to represent dominant erosional velocities. Scallop-forming discharges are tentatively derived through the Manning equation, and although the results show low flows to be of importance in swallet streams, and higher stages in percolation streams, the attendant calculation of depth is entirely unrealistic.

Markov models are investigated and found to be of form significance, especially in the changes occurring in transformation. Spectral analysis is rejected in terms of classical methods, but found helpful in comparative studies. Cave meanders show an increasing sinuosity and complexity, and a downstream translation of bends during evolution.

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TABLE OF ABBREVIATIONS

1. Cave Passages Surveyed

(a) Ireland

Mains	Mainstream Poulnagollum: 149 1.5m legs
Shaft	Shaftgallery, Poulnagollum: 135 1.5m legs.
Culla	Cullaun I: 125 1.5m legs
Caths	St. Catherine's Cave: 101 1.5m legs.
Polld	Polldonough, Coolagh River Cave: 84 1.5m legs.

(b) New Zealand

Hole-L	Upper Gardners Gut, downstream from Hole 'L' inlet: 252 1.5m legs.
Hole-H	Same reach as Hole-L, but surveyed some 2m above: 232 1.5m legs.
Hole-N	Hole-H, corrected by closure onto Hole-L
Hole-E	Continuation of Hole-L into area of more complex fracturing: 128 1.5m legs
Hole-J	Continuation of Hole-E, but demonstrative joint control: 97 1.5m legs.

2. Transformations

B	Original bearing series.
D	Deviation from the mean bearing series.
dB	Curvature, or change in direction series.
ddB	Change in Curvature series.

3. Individual Bend Properties

Meandb	Mean curvature
Sdevdb	Standard deviation of curvature
Bendsp	Bendspacing
Sinuos	Sinuosity
Arclen	Arclength
Orient	Orientation
Meanwi	Meanwidth
Sdevwi	Standard deviation of width

CHAPTER I

INTRODUCTION

The task of formulating laws in geomorphology does not lend itself to precision, for it is easier to generalise on the nature of the landscape, than to precisely define its component forms and processes. River meanders present a possible exception to this argument, for they are widespread in their occurrence and apparently regular in their form. Such homogeneity implies that a law of some kind governs their form. The search for such a law has so far succeeded only in illustrating the complexity of fluvial processes and the multivariate nature of the problem.

Flowing water meanders on a wide range of substrates; alluvium, bedrock and ice. Outside the constraints of a confining channel, the Gulf Stream and small trickles of water both exhibit a meandering form.

The present work is concerned with the apparently meandering form of underground stream passages in limestone caves. These have been observed in both vadose and phreatic passages,[1] although the latter are not considered specifically here.

Cave streams develop through the interaction of both solutional and mechanical erosion, and their erosional processes, therefore, may be drawn from both the bedrock and supraglacial cases. The bedrock environment is heterogeneous, both in lithology and fracture distribution, and the constraints this imposes on meander form are unknown.

Cave meanders are unique, in that as downcutting proceeds earlier channel forms are preserved in the walls of the cave. It is therefore possible to determine the change in form over (relative) time and to study the evolutionary pattern of the meanders.

Finally, boundary-layer turbulence between aggressive (corrosive) water and limestone produces a mosaic of scallop forms on the rock. The scale of these forms is inversely related to water velocity. This allows some tentative hydraulic interpretations to be made concerning cave streams.

The purpose of this thesis is to consider the above mentioned problems in cave geometry and to investigate these features in the context of previous work on stream meanders.

Footnote:

1. See Monroe (1970) for a glossary of karst terminology.

(All footnotes are located at the end of the appropriate chapter.)

CHAPTER II

THE CONTEXT OF THE THESIS

There has not yet been a logical account of why rivers meander and any formal explanation seems contingent on an improved understanding of fluid mechanics. Instead, research has been addressed to the problem of how a river meanders; such as description of channel form, and the nature of processes observed in meanders. This provides a valuable data base for the evaluation of subsequent explantions of meandering.

(i) Planform Description

Bates (1939), Chately (1940), Prus-Chacinski (1954) and Chitale (1973) described meandering channels as combinations of circular segments, a method which may be supplemented by the use of linking tangents (Ippen and Drinker 1962, Yen 1967). Leopold and Wolman (1960) and others have suggested a sinusoidal approximation to river form. This was improved upon by Langbein and Leopold's (1966) proposal of the sine-generated curve. The distribution of curvature elements along a river channel was assumed to be normally distributed. The model was associated with a random walk concept of meandering and has been important in the statistical school of river description, which is discussed later.

Ferguson (1973b, 1976) has compared several mathematical models for meander planform, drawing on one parameter for scale and one for slope. He found the models appropriate only for individual meanders and unable to encompass natural variability in form unless some random disturbance was added to the model. The sinuosity of direction variance of a river channel is ambiguous in isolation, but Ghosh and Scheidegger (1971) proposed the degree of wiggliness, which allows a more complete description of planform. The characterisation of river planform is possible using spectral techniques, the methodology of which is discussed below.

(ii) Hydraulic Geometry and Regime Analysis

The term regime here refers to the relationship between measurable variables of a river in equilibrium or whose "...average behaviour does not change greatly over small periods of historic time" (Blench, 1957). It is not to be confused with "regime" meaning variations in flow with time (Chitale, 1976). Regime analysis relates to the behaviour and design of artificial channels such as irrigation canals (eg. Kennedy 1894, Lacey 1929, 1933, Inglis 1941, 1949).

The natural equivalent of a regime canal is a river in equilibrium. The form of a river is described by its hydraulic geometry. (Meander planform parameters are defined in Figure 1.) Studies of hydraulic geometry (eg. Leopold and Maddock 1953) are concerned with the relationships between hydraulic and geometric variables. Leopold et al (1964) employed the method to characterise the downstream behaviour of individual rivers and tc compare these properties for different rivers and regions. The findings of both schools have been generally compatible and are treated as such here.

Bates (1939), Inglis (1947) and Leopold and Wolman (1957, 1960) found correlations between the scale (wavelength) of meanders and stream width, radius of curvature, and discharge. Width is not independent of discharge which is the major independent hydraulic variable (or catchment area, its surrogate), although width is far more conveniently measured.

Slope has been looked upon as an independent variable (Friedkin 1945, Carlston 1965, Richards 1972), although erosion and aggradation are clearly capable of altering its value in a sedimentary system. (The meaning of independence is itself subject to time-scale constraints, Schumm and Lichty 1965.) Schumm (1967), using field examples, and Ackers and Charlton (1970a,b,c) working with a large flume, found gradient unimportant in determining meander scale, although stream cross-sectional geometry has been related to slope (eg. Park 1976).

The other major independent variable is material (Schumm 1963, 1967), and its cofactor, vegetation. Smith (1976) noted the additional strength imparted to noncohesive bank material by vegetation. Schumm found that streams with a high silt-clay content in their sediment load generally exhibited smaller meander wavelengths than streams carrying a larger calibre of material. The two types are divided by the overall best-fit regression line. This implies that erosional and transportational processes may be a major mechanical factor in determining meander scale.

(iii) The Problem of Variables

A major problem in the field measurement of geomorphic phenomena is their inherent variability in time and space. This is generally dealt with by assuming random variability about some measure, usually the mean. A large sample is desirable because the sample parameters must generally be assumed to be those of the parent population. Data are seldom normally distributed and logarithmic transformation is common.

Sampling problems are both practical and statistical. It is generally impossible to achieve experimental control in a situation and this results in multivariate problems. Also, in gathering a large sample, problems of stationarity (Yevjevich 1972) and spatial dependence (Cliff and Ord 1972) are encountered. These problems are commonly ignored in the interests of obtaining a large volume of data.

If a variable is to be assigned a finite value (ie. become a constant), it is implied that this value is representative. This process often ignores the higher moments of the data and any polymodality therein.

Langbein (1963, in Leopold et al 1964, p271) has classified certain hydraulic variables into dependent (adjustable) and independent (given) categories. Although empirical relationships, such as power laws, may be applied to these variables, no substantive interpretations can be made. For example, the equations are often dimensionally unequated (eg. Table 1). Problems associated with some of these variables are briefly reviewed below.

(a) Discharge

Discharge is a major variable component of the fluvial system and its description depends largely on the context in which it is employed. Jefferson (1902) used the mean annual discharge, but later work sought some morphologically significant discharge where ..." equilibrium is most closely approached and tendency to change is least. This condition may be regarded as the integrated effect of all varying conditions over a long period of time" (Inglis 1947). There are no logical, temporal criteria for defining a morphologically dominant discharge. Attempts to define a dominant discharge in flow-duration terms have been made. Stall and Folk (1968, in Daniel 1971) found that the 10% flow duration was most appropriate in explaining hydraulic geometry. Daniel (1971), on the basis of unsupported travel-time considerations, used all discharges above average to define channel-forming discharge

The absence of any theoretical basis for defining dominant discharge, along with difficulties in the application of controlled regime work to natural, temperate conditions, led Nixon (1959) to determine the frequency of bankfull discharge in Britain. Bankfull discharge was assumed to represent the optimum discharge in a channel, while both overbank and low flows represented a state of disequilibrium. Nixon determined a recurrence interval of six months for such a flow. He was careful to point out the regionally-dependent nature of variables such as sediment transport, which rendered his work geographically specific. Leopold, Wolman and Miller (1964) found a recurrence interval of one to two years for the bankfull discharge of American rivers. Their conclusions were based on a wide scatter of data and any interpretations must be correspondingly general (Kennedy 1972).

Carlston's (1965) review of "... the relation of free meander geometry to stream discharge..." attempted to determine the most suitable discharge parameter from a series of regressions. A discharge between mean flow of the month of maximum discharge and the mean annual discharge proved most suitable. He noted a hysteretic effect in which a falling stage flow was more important in erosion than the same flow at rising stage, due to increased weakness following wetting of the bank. His definition of dominant dicharge is important as it may possibly be valid in a range of climatic regions and, therefore, could accomodate a wider range of data.

Ackers and Charlton (1970a,b) developed earlier experimental work in flumes (eg. Tiffany and Nelson 1939, Matthes 1941, Quraishy 1943, Friedkin 1945, Ackers 1964), investigating the importance of various parameters under controlled conditions. By establishing discharge-meander wavelength relationships under steady flow, they were able to determine the dominant discharge for the finite meander forms developed under conditions of varying flow. This essentially extended Blench's (1951) definition of dominant discharge as "... the steady discharge that would produce the same result as the actual varying discharge...", to flume procedures. Bankfull flow was found to be the dominant discharge. It was argued that meander wavelength was the most conservative feature of the river channel, compared to depth, bedforms, or width, and thus it was a response to some long-term measure of flow. The pattern of discharge and the sediment regime of the river in question was also emphasised.

These findings confirmed those of Wolman and Leopold (1957) who had concluded that within-channel processes were more important than overbank processes, because floodplain sediments were generally not those believed to be derived from overbank floods. Similarly, Wolman and Miller's (1960) statement that flows less than overbank were dominant in fluvial geomorphology was endorsed. However, Stevens et al (1975) claimed that in certain disequilibrium rivers, channel alteration was directly related to the magnitude of a particular flood event.

In alluvial meanders, adjustment of form is achieved by the redistribution of channel sediment. This, in addition to the difficulties inherent in defining bankfull or dominant discharge (Nixon 1959, Kennedy 1972), led Benson and Thomas (1966) to suggest that dominant discharge be defined as "...the discharge that over a long period of time transports the most sediment." Discharges calculated in this manner were found to have a recurrence interval of some 1.5 months, which was substantially more frequent than previous definitions of dominant discharge. A major criticism of this definition is that basin behaviour as a whole is monitored (see Kirkby 1971) and a large part of the suspended load would be derived from outside the river itself. Material defining river form is eroded from banks and bed and is frequently transported only to the point bar immediately below the point of entrainment (Friedkin 1945, Leopold and Wolman 1960, Hooke 1975).

Blench (1957) had suggested a similar definition of dominant discharge based on bedload. Pickup and Warner (1976) and Pickup (1976) calculated that most bedload would be moved by discharges with a recurrence interval of from 1.1 to 1.5 years, although D.I. Bray (1975) considered two years more appropriate. It was noted by Pickup and Warner that discharges responsible for bedload movement were not important in bank erosion, which was effected by much larger floods. They pointed out that, over time, different dominant discharges are probably responsible for maximum rates of different channel processes. It therefore appears that dominant discharge is also a complex quantity, whose characteristics are determined as much by channel properties as by the flow variability.

Cave streams display dissolutional markings, which have an established relationship to flow velocity. This may allow an estimation of dominant erosional flow rate in a solutional environment.

(b) Wavelength and Sinuosity

Assuming an appropriate definition of discharge, the nature of the river's meandering form needs defintion. Jefferson (1902) used meander belt width, which has subsequently been found independent of flow criteria (Carlston 1965, Ackers and Charlton 1970b). Later work (eg. Lacey 1933, Inglis 1947) employed the downvalley wavelength of meanders, a parameter which has since been widely used (eg. Leopold and Langbein 1966). Carlston (1965) pointed out the difficulties inherent in the field measurement of meanders and the subjectivity involved in selecting a "free" or well-formed meander, especially where exceedingly irregular and tortuous meanders occured.

Mackay (1963) suggested the use of spectral analysis in meander studies and Speight (1965a) employed the technique, recommending it on three premises. He considered that subjective choice of "free" meanders might bias results and that the arbitrary linear definition cf wavelength was meaningless. Furthermore, the possibility of multiple wavelengths (eg. Jefferson 1902, Hack 1965) was precluded. (Spectral procedures are described later.)

The application of spectral analysis (Speight 1965a, 1968) indicated that meander wavelength was indeed polymodally distributed, and therefore could not be characterised by a mean value. Chang (1969), Church (1972) and sperare:beth (1974) also applied spectral techniques for meander planform description. Meanwhile, Yalin (1971, 1972), using considerations of turbulence, had predicted the existence of multiple meander wavelengths related by multiples of two pi. Hey (1976) has suggested that four pi would be a more reasonable estimate for the factor, although Parker and others (1976 pers. comm.) have questioned the theoretical validity of Yalin's work.

There remain several problems in the use of spectral analysis. For example, long series of data are necessary which, nevertheless, must be drawn from an homogeneous section of river without any major changes in discharge, material or environment along its length (Chang and Toebes 1970). This is a type of stationarity.[3]

The technique of spectral analysis produces results as "bands" or class ranges rather than as finite values. These bands are in the frequency domain, although they are easily converted to the more familiar space domain. These problems, together with difficulties in dealing with several wavelengths, tend to result in conceptual and comparative difficulties (Jenkins 1961, Speight 1965b, Curl 1972 pers.comm.).

Furthermore, spectral analysis demands sequential, progressive and, more usually, equal-spaced sampling of data values. It is impossible to represent most river meanders in this way as simple spatial coordinates. Generally, meanders have been discretised into a series of orientation values of sequential, equal-length segments along the channel thalweg or centreline. The effect of this transformation from the cartesian system (if any) is obscure and it has only been explicitly recognised by Surkan and van Kahn (1969) and Rieger (1976 in abstract). The latter investigated the problem and concluded that it resulted in wavelengths being calculated along the channel. Cross spectral analysis of the X and Y Cartesian co-ordinates gave a similar channel-length value for wavelength, as well as indicating the displacement of the bend from the mean flow direction. The data analysed are generally first order differences of the raw series. This is the curvature series for the channel and acts as a high-pass filter, which removes major 'topographic' trend from the data.

Apart from practical problems involved in discretisation, such as information loss (Dyhr-Nielsen 1972) and aliasing[2] (Jenkins 1961, Gunnerson 1966), spectral analysis is a sine and cosine transformation of the autocovariance of a series. The relationship between the wavelengths determined by more conventional techniques and those obtained by spectral analysis is unknown, but has been assumed to be interpretable (Speight 1965a, Ackers and Charlton 1970c). Ferguson (1975) did find the relationship between discharge and dominant spectral peak to be similar to that determined from conventional studies.

Hickin (1974 p440) pointed out that spectral analysis considered the total oscillatory behaviour of a river and, as such, was unable to filter naturally occurring noise. He went on to say:

> ...the size of channel bends is often influenced as much by the nature of natural migration as by the...meander wavelength.... Power spectral analysis provides the only effective way to describe the oscillatory behaviour of a meandering channel, but using it to define a simple index of the meandering scale is an abuse of the technique.

Other descriptions of the degree of meandering, such as sinuosity have been widely used. Sinuosity may be defined as the stream length over the linear distance between two points; the topographic sinuosity (Schumm 1963). Other workers (eg. Brice 1964, Mueller 1968) have used the hydraulic sinuosity, which is the average of a number of topographic sinuosities from short lengths of channel and which removes effects of valley curvature (Haggett and Chorley 1969).

Attempts to classify rivers on the grounds of sinuosity (eg. Schumm 1963) have not been very successful due to the ambiguity of sinuosity (Hey 1976). Ghosh and Scheidegger

(1971) proposed the parameter "degree of wiggliness", a measure of planform supplementary to sinuosity. It is based on curvature and, unlike sinuosity, can be used to distinguish between intensely meandering rivers with generally small wavelengths from large amplitude meanders.

It should be noted that discretisation shortens stream length, which reduces sinuosity. The effect is related to the magnitude of the absolute curvatures. Chang and Toebes (1970), Thakur and Scheidegger (1970) and Ferguson (1977) pointed out that the variance of a series is also a measure on the sinuosity. The variance of directional data should be calculated as one minus the mean vector strength (Mardia 1972), but linear methods have been generally used (eg. Ferguson 1977). This has drawn attention to the variability of sinuosity with direction dispersion, when in fact a precise relationship exists.

(c) Other Variables

The slope of either the channel or the valley is generally considered as overall or average slope. It has been established that slope has an important bearing on stream form (eg. sinuosity, although apparently not meander scale). Pools and riffles, straight and meandering, and meandering and braided streams are distinguishable using slope criteria (Leopold and Wolman 1957). Some authors consider gradient an important variable (eg. Friedkin 1945, Carlston 1965, Ferguson 1973a), while others have declared it unimportant (eg. Schumm 1967, Ackers and Charlton 1970b). Dury (1973b) has pointed out that sinuosity can be related to valley gradient, an adjustment affecting stream gradient.

Width and depth have been generally reduced to mean values, although both are known to vary downstream in a more or less complex manner which depends upon location within a meander (Brush 1961, Dury 1976a), sinuosity (Mackin 1956), and material, as well as discharge (Knighton 1974, Richards 1976a).

Greater cohesion of bank material (percent silt-clay) has been found to increase the sinuosity of rivers (Schumm 1963,1967). The effect of bedrock on meanders is discussed below. Although a major independent variable, the influence of material on river form is poorly understood.

It has been implicit in studies of rivers that concealed within complex patterns are simple rules. This means most studies have concentrated on the reduction of data into the form most easily manipulated. Attempts to comprehend the natural situation require complex methodologies and sympathetic recognition of the variables involved.

(iv) Meander Evolution

Fluvial, floodplain deposits are derived from overbank flooding and point bar growth. Studies of floodplain material indicate that the latter is generally the more important component (Wolman and Leopold 1957, Lewin and

Manton 1975). This implies that point bars and meanders shift their locations through time, a fact necessarily attendant on the differential distribution of erosion and deposition within the meander bend.

The regime and hydraulic geometry analyses suggest that this migration is accompanied by maintenance of form which, at least in an historical context (Schumm and Lichty 1965), is time independent (eg. Leopold and Wolman 1960). Flume analyses have shown a maintenance of form following initial development and an orderly, downstream translation (Daniel 1971) of the meanders (eg. Ackers and Charlton 1970b). Hooke (1975) has explained this in terms of shear stress distribution in bends. Tiffany and Nelson (1939) observed an increase in amplitude of flume meanders over time, although the rate of growth gradually declined. Continued increase in amplitude, and ultimately meander cutoff have been considered an effect of inhomogeneous bank material (Friedkin 1945, Leopold et al 1964). Daniel (1971) noted that migration rates were somewhat inversely proportional to the ammount of silt-clay in bank material, and therefore proportional to width-depth ratio (Schumm 1960). This is confirmation of the importance of material strength as well as applied stress in form evolution.

Many natural rivers, however, exhibit little consistency of form (eg. Schumm 1963). There is debate as to whether this is a result of polymodality of form parameters (Speight 1965a), floodplain heterogeneity (Fisk 1952, Ferguson 1973b), or natural characteristics of river flow (Yalin 1971). The flume analyses are not of a scale, nor duration, to answer this question.

The majority of work on meanders has considered them to be time-invariant. However, Lewin (1972) has described an increasing complexity of form attendant on meander growth, which he attributed to pools and riffles maintaining a regular spacing along the channel during meander evolution. Ferguson (1973b), however, argued that the evolution of meander form was itself time-dependent and that at later stages a statistical equilibrium was reached where time was less important. The assumption of stochasticity is considered below, and polymodality has been discussed.

The study of the natural evolution of river meanders requires an historical record of earlier river forms. Speight (1965a,b), using two series of aerial photographs some thirty years apart, determined that the spectral characteristics of rivers did not change with time, although he observed no orderly migration of meanders. He noted stream cutoffs, but did not imply (eg. Strahler 1946) that this process was responsible for maintenance of scale. Parker (1976 pers. comm.) has suggested that meander growth is ultimately limited by the cutoff mechanism, although the rate of meander arc lengthening declines over time as a result of lessening slope.

Brice (1974) constructed a set of models for meander evolution based on circular arcs and linking tangents. He defined several forms exhibiting both symmetrical and asymmetrical evolution. He described a general tendency for the radius of a loop to decrease as amplitude increased, to a point where linking tangents reached some critical length, whereupon subsidiary loops developed. This characteristic may be a cause of polymodal meander wavelength.

Hickin's (1974) work confirmed the above descriptions of meander evolution. He reconstructed old channel form for a series of bends by mapping scroll bars from aerial photographs. The points of maximum erosive activity (migration) were located. After an initial period of growth perpendicular to the initial channel, downstream migration of the zone of maximum erosion became increasingly important. The outward growth resulted in a sharpening of the bend, until the ratio of radius of curvature to width (Rc/w) fell to a value of just over two, at which point erosion was curtailed. The local form then exhibited an apparent equilibrium, and zones of maximum erosion developed elsewhere (generally downstream). When several of these sequences had occurred the lengthening of the bend might be such that two or more areas of rapid erosion would develop, producing compound loops. There was also a tendency for the growth of a particular bend to be matched by the shrinkage of an adjacent one. This indicates that the lowering of slope by meander growth may to some extent be rectified by compensatory shrinkage elsewhere, although it appears a shrinking meander bend has never been explicitly documented.

Employing dendrochronological techniques, Hickin and Nanson (1975) established migration rates for channel bends. It was found that as Rc/w decreased, the migration rate (ie. erosive activity) increased. This reached an implicit (no examples were recorded) maximum at a ratio of three and then declined dramatically at lower values.

Hickin's findings were accommodated within experimental and field observations. Studies of flow resistance in bends (eg. Bagnold 1960, Leopold et al 1960) showed a minimum resistance to turning at Rc/w values around two. The shear stress in a bend shows a gradual increase as Rc/w is decreased. This, Hickin inferred, was manifested in higher erosion rates as curvature increased. The greater erosive force and superelevation contingent on acceleration are claimed to reduce relative roughness and, therefore, flow resistance around the bend. However, tighter radius of curvature also requires more internal deformation of the water. This reaches a critical level at a ratio of two, where spill resistance is initiated and turbulence generated. This dramatically increases the flow resistance.

The widespread observation of meander bends exhibiting Rc/w ratios of around two to three (eg. Leopold and Wolman 1960) was invoked by Langbein and Leopold (1966) in their theory of minimum variance. (Hey, 1976, however, observed that this ratio was not as common as implied by these authors.) They proposed a form model based on the sine-generated curve which complied with natural Rc/w ratios. It was suggested that a river following this route expended the minimum energy in turning.

Daniel (1971) suggested that the yearly variability of migration rate might be related to flood frequency and, therefore, dependent on the climatic environment.

(v) Statistical Analysis

In the empirical approach to meanders, consistency can be obtained through either the subjective control of sampling, or by the use of controlled experimental methods. Theoretical models may also be developed (eg. Ferguson 1973) An alternative philosophy is to assume that river form is too complex for deterministic explanation and that statistical models are appropriate.

(a) Statistical Models

Harvey (1969, p260) described possible philosophical grounds for the use of probabilistic methods. Generally, when a particular situation produces variable results (in either controlled or field conditions) it is sometimes difficult to account for this variability explicitly. It is often more convenient to consider the situation from a probabilistic point of view. Many statistical tests are based on the assumption that measured values are derived from the "parent population" via random selection. As the sample size increases, the sample frequency distribution will approach the distribution of its parent population.

Random selection may be possible in an experimental situation, and although geographical field measurements may be sampled randomly, the phenomenon may not itself be randomly distributed. As Hepple (1974 p94) pointed out...

> Near places are on the whole similar- a truism that Tobler has expressed as the first law of geography: "everything is related to everything else, but near things are more related than distant things..."

This means that the assumption of independence of data points demanded by most commonly used statistical tests is not met by geographic data.[3] However, arrangement into a frequency distribution can assuage this problem so that tests may be made on the data (Kelker 1976 pers. comm.).

The statistical treatment of meander data is contingent on the form in which the data are gathered. The regime and hydraulic geometry methods produce data as sets of variables gathered at specific locations on a river or canal. These data have been appropriately analysed using regression techniques, ie. while some variability (error) is anticipated, the relationship between two variables is considered to be ultimately deterministic and straightforward, and therefore manifest in a suitable regression equation. This is a valid use of the technique, providing the assumptions regarding the functional relationship and dependence of the data are correct.

Regression analysis seeks an underlying relationship between phenomena, but it is only an exploratory tool in research, because it only indicates a numerical relationship between variables, and has no functional (deductive) significance. The non-deductive (often called inductive) approach to investigations is essentially concerned with identifying possible controls over a given situation. It can never prove, only support a relationship. However, statistical methods do provide powerful, objective methodologies for dealing with situations of uncertainty.

Meander data may also be gathered from long reaches which span more than one bend. Here the stream planform is generally discretised into a series of directions of contiguous, equal-length segments along the stream centreline or thalweg. These data are generally dominated by low frequency, valley form components which may be filtered by differencing. This is equivalent to prewhitening with a coefficient of unity (Rayner 1971). Data gathered over a given reach may then be described by their distributions and compared to other series, or expected distributions. For example, Chang and Toebes (1970) found the distribution of curvature (first difference of direction) elements from meandering rivers were affected in variance and kurtosis by geological and discharge factors.

Most statistical studies of meander series have been empirical. However, Langbein and Leopold (1966), drawing on established random walk theory (von Schelling 1951), suggested possible physical reasons for the normal distribution of deviations in meanders. This finding was confirmed by field investigations (Langbein and Leopold

1966, Thakur and Scheidegger 1968), although the problems of independence (Hepple 1974) and stationarity (Chang and Toebes 1970) were not considered. Scheidegger (1967) also demonstrated a thermodynamic model of river meandering in which the meander sinuosity could be considered analogous to "temperature". Thakur and Scheidegger (1968) determined the values of temperature for several reaches, but they made no interpretation of the physical or statistical significance of these values and their observed variability along river lengths.

Mathematical modelling of meanders was found successful only in accounting for single bends, and frequently some unexplained variability was encountered (Ferguson 1973b). Analogue models employing a necklace chain were considered to mimic the form of a meandering river and a lunar "rill" (Thakur and Scheidegger 1970). The clear relationship between the orientation of successive links of the chain suggested a first-order autoregressive model. A subsequent study of "natural wiggly lines" found similarities in the form of meanders, crenulated divides and erosional coastlines (Ghosh and Scheidegger 1971), but the similarities were not compared in terms of statistical criteria. It appears that these statistical descriptions of meanders do not have any substantive significance.

The meander model derived from the random walk by Langbein and Leopold (1966) produced a geometry characterised by the sine generated curve. Surkan and Kan

(1969) attempted to generate meander forms under the statistical constraints of the sine generated curve and the simulations were unsuccessful, even when a "random component" was added. Both autocorrelation models and transition parameters indicated serial dependence, which is characteristic of Markovian models. Meander forms were found to exhibit a directional bias in variation of curvature (change in direction) components. The further a series deviated from its mean direction (presumably valley slope), the wider a range of changes in direction was found. Models generated under this further constraint were found to more closely resemble natural meander forms, although Ferguson (1976) did not find this property in meander series. Surkan and Kan's subsequent testing of model properties, however, would appear to be implicitly tautologous, because these properties were largely imparted by the initial controlling conditions for the model.

The sine generated curve has, nevertheless, remained a widely accepted meander model (eg. Hey 1976), but Ferguson (1976) criticised its fundamental derivation and suggested a second order autoregressive modelling procedure based more on process considerations. His disturbed periodic model assumed the meandering phenomenon to be fundamentally regular, but the natural heterogeneity of the floodplain was seen to induce disturbances, the effect of which was gradually lost downstream. The attempt to interpret real meander forms in terms of the model remained somewhat generalised, however.

Ghosh and Scheidegger (1971) found that natural wiggly lines could be defined on the grounds of sinuosity and "wiggliness"; a quantity defined by the average of the sum of squared deviations, and in some ways related to "temperature".

The findings in the field to date have rarely attempted to rationalise the data in terms of geomorphological processes or the regime and hydraulic geometry approaches. Attention has been directed more to the model, than its real significance.

Discretisation has received some attention as regards information loss (Dyhr-Nielsen 1972), as have the problem of selecting leg length and sampling interval (Gunnerson 1966, Ferguson 1975). The loss of information through averaging (ie. taking direction between two points) rather than point measurement of direction is perhaps unimportant, although some more extreme values may be lost. Thalweg measurements are most suitable if meander form is to be fully described. The more commonly used channel centreline probably smooths the series by lessening bend amplitude. Ferguson (1977) related direction variance to sinuosity and compared the results from rivers with a sine generated curve. The fixed relation of sinuosity and (circular) variance means that the difference between the results can only be due to the treatment of the sine-generated curve as a continuum, while the river data were discretised.

Leg length has usually been selected arbitrarily, without consideration of physical effects. While error analysis can provide suitable criteria, there has been scant attention paid to the possiblility of aliasing² in practice, or to the effect of varying sampling interval. Discretising shortens a series to an extent dependent on the periodicity and amplitude (see Richardson 1961) and can produce series with radically different statistical properties (see Thornes 1973). Ferguson (1975) suggested that river width provided an appropriate objective leg length. This could be useful in combatting problems of stationarity, given the correlation between width and meander size, which is manifest in the observed scale independence of meander form. There remain the problems of the non-monotonic (perhaps periodic) increase in width downstream and the only roughly linear relationship between width and meander wavelength.

The general problem of stationarity has proven a major constraint in analyses. Stationarity is important in both statistical procedures and substantive interpretation. The latter is concerned with the degree of consistency along the channel of factors, known 'a priori' to influence meandering. Long series, preferable from the statistical point of view, are severely constrained by this problem. Linear trend may be removed by taking residuals from a linear regression. Lack of stationarity of higher moments presents less tractable problems. A stationary series may converge* on its parent population only as sample size increases. The samples encountered in geography are often small and possibly trivial. It is neither possible to envisage them as stationary, nor to test for more than weak stationarity (Yevjevich 1972), as single samples are the rule. Under some circumstances, prewhitening or differencing can help reduce non-stationarity in series which exhibit strong memory (Granger 1975).

It appears, however, that in practical applications it is possible to proceed with analysis of data known to be non-stationary to some degree. Tukey (1961) said

> ...I have yet to meet anyone experienced in the analysis of time series data...who is over concerned with stationarity. All of us give some thought to both possible and likely deviations from stationarity in planning how to collect or work up data, but no one of us will allow the possibility of non-stationarity to keep us from making estimates of the average spectrum....Once we admit we are making an average spectrum, we have admitted that there may well be other relevent characteristics of the situation beyond the spectrum.... Such an admission...is a good thing rather than a bad one.

Statistical analysis can proceed on any series, however, it must be acknowledged that non-stationarity will result in average descriptors which are not necessarily representative of any part of the series. It is perhaps more important to recognise physical constraints in geomorphological situations, for no amount of statistical exactitude and precision can compensate for inconsistencies in the designation of the original series. Watson (1966 p786) emphasised that "...the most important part of the analysis precedes the application of these [statistical] methods."

b) Stochastic Models

There has been considerable attention paid to the analysis of series in mathematics. The procedures of this analysis have proven valuable in the description and evaluation of geographic data. The variability of a geographic series can be considered to be generated by stochastic processes, which are concerned "... with systems which develop in time or space in accordance with probabilistic laws." (Cox and Miller in Hepple 1974).[5] Two major branches of the stochastic approach are of present concern: spectral analysis (Jenkins and Watts 1968, Rayner 1971) and Markov chains (Kemeny and Snell 1967, Harbaugh and Bonham-Carter 1970).

Some practical aspects of spectral analysis are considered elsewhere (section iii,b). The spectrum of a series essentially describes the contribution of groups of frequencies of sine and cosine functions to the overall variance of a series. The spectrum is usually computed as the Fourier (sine and cosine) transformation of the autocovariance of a series (although other methods do exist, eg. see Edge and Liu 1970, Rayner 1971). Covariance is a measure of the common variability between two series, and autocovariance describes the common variability of a series compared to itself when offset at successive lags. The autocovariance at the initial lag (zero, when each point is compared to itself) is the variance of the series and successive lagging will produce lower values, depending upon the strength of the similarity between successive points in the series (unless the series is perfectly periodic). Oscillations of the autocovariance may be indicative of periodicity, although the spectrum is more readily comprehended. As the spectrum is only a transformation of the autocovariance (or, in the case of the normalised spectrum, autocorrelation, which is the autocovariance over variance) and contains no more information (Chow and Karelotis 1970); tc compare periodicities determined from both sources (eg. Ferguson 1975) is a redundant procedure.

The spectral estimates run from frequency zero to the Nyquist frequency[6]. The extent to which a series is lagged depends on the resolution required and the relative loss of information from the series through lagging. Generally, between ten and fifteen percent of the number of sampling points is chosen (Jenkins 1961, Brown 1972).

Problems in the interpretation of spectra have been discussed, but it is necessary to add that, since spectral analysis is a non-parametric (distribution free) technique "...several processes [distributions] may give rise to the same spectrum." (Hepple 1975). However, one major purpose of spectral analysis is to identify the presence of different frequencies which may stem frcm different underlying causes. The presence of a particular frequency may not necessarily stem from a periodic component and peaks may be propogated as "...echo effects down the spectrum" (Kendal 1973).

Applications of spectral analysis to meander studies have been of only limited success, at least in terms of the substantive interpretation of the results. Chang and Toebes (1970) observed different spectral characteristics for geologically distinct reaches of a single river system and Ferguson (1975) established relationships between the dominant spectral peak and both width and discharge. The latter author did not observe several spectral peaks as did Speight (1965a, 1967) and Chang and Toebes (1970). The spectra generated by Thakur and Scheidegger (1970) for a chain analogue of river meanders were different from those reported for rivers by other authors (as well as themselves), despite claims of similarity. Subsequent use of the same model (Ghosh and Scheidegger 1971) gave substantially different results

The distribution of events through time or space can often be successfully modelled using stochastic processes. The entirely random or independent situation is governed by Bernoulli processes (Whitelegg 1976). Where the probability of an "event" occurring increases only with time (or distance) from a preceeding "event", a Poisson model is appropriate (or binomial for high probabilities of occurrence). Where the probability of a state is conditioned by a finite number of preceeding events, the series may be described by a Markov process.

If the last type of series can be classified into states, then a discrete Markov model is appropriate. The series is considered in terms of the transitions from one state to another. If these transitions are clustered into a square matrix whose order is the number of states, then the overall proportion of transitions of each type can be calculated by dividing elements of the matrix by the total number of transitions. The result is the transition proportion matrix (TPM, see Fig. 2). If the matrix elements are divided by their row totals, the matrix is called the transition probability matrix, which shows the probability of the next state, given the present state, as opposed to the overall distribution of states through the series. It has been widely used in stratigraphic modelling (Harbaugh and Bonham-Carter 1970).

Contiguous segments of river meanders may be readily classified into two states, positive and negative, either by taking the signs of the deviations from the mean direction, or by differencing (taking derivatives). The characteristic dichotomous TPM can then be constructed for a given series (see Fig. 2). Surkan and Kan (1969) first employed the TPM and found it gave useful insights into meander form. They recognised a tendency for river meander TPM's to remain in a particular state, which decreased with differencing. This implied that at least directional and first difference (curvature) series were not serially independent, as had been assumed. (Independence would be indicated by a symmetry between the correlation and anticorrelation diagonals, see

Fig. 2 for definition of terms.)

Early interpretations of the TPM (eg. Thakur and Scheidegger 1970, and Ghosh and Scheidegger 1971) were unfortunately characterised by ambiguity or misinterpretation of the matrix and a failure to recognise the problems and benefits inherent in the the use of the matrix. The fundamental relationship between the TPM and Markov chains has not been recognised. There have also been no studies which test for the memory implicit in the matrix; a TPM can be generated for any series and is not necessarily a significant predictor if the Markov property is not present. Geomorphology has seen some use of Markov chains (eg. Thornes 1973 and Shaw 1975), but they have been used more widely in other fields (eg. Rodriguez-Iturbe et al 1971 in hydrology, Collins 1975 in human geography, and the work in stratigraphy, eg. Harbaugh and Bonham-Carter 1970). The need for proper caution in the use of Markov models has been stressed by Lloyd (1974) and Whitelegg (1976).

The Markov chain approach has promising potential, but care must be taken to distinguish those effects produced by data manipulation from those of substantive significance.

c) Controls on Cavern Form

The meanders observed in caves (which are more fully discussed later) may be analysed in several ways. Ongley (1968) used simple descriptive statistics, and Deike and White (1969) used regression. Cave form can be controlled by fractures in the bedrock and statistical methods appear intuitively suited to consideration of this problem.

Many cave passages are entirely subordinate to joints and this has been qualitatively demonstrated by R.Deike (1967) and Weaver (1973) using rose diagrams. G. Deike (1967) and Ogden (1974) attempted to use non-parametric tests to compare distributions of fractures and straight cave segments. They found no significant control, although correspondence was obvious in some locations. These authors emphasised that joint types are not homogeneous in their effect on the stream passage, a result of different joint types and different hydraulic conditions. Therefore bedrock joints and fractures do not constitute a simple population and can be considered neither a controllable, nor an independent variable.

There is a wide range of tests applicable to the directional and axial case (Mardia 1972, Watson 1966). However, the lack of familiarity with the characteristic distributions and statistical premises perhaps accounts for the scarcity of geographical applications.

Ongley (1968) considered joint control would be manifest in the distribution of passage direction. If no structural control were acting, then a Poisson distribution of the frequency of occurrences of zero, one, or more axial bearings per degree class might be expected. The method is ingenious, but was limited in its application by too small a sample. It is also questionable whether one degree intervals are appropriate, because in an oscillating system, passage orientation may be responding to joint control, but still exhibit some small departure from the precise joint direction.

In conclusion, the problems of non-deductive methods must be recognised. Assuming that results can be readily interpreted (which they can not), the techniques are often not wholly applicable to geographic data. While statistical methods do offer powerful methods of dealing with variability, care has to be exercised in their application. The statistical technique can only be justified by its results and is not an end in itself.

(vi) Process Analysis

There is little agreement on the source of the meandering processes, or indeed on what those processes are, but the literature on processes in meanders is briefly reviewed.

(a) The Coriolis Effect

The coriolis force (or effect) has been widely invoked as responsible for meandering, for example by Gilbert(1884), Eakin(1910), Chately(1938) and Quraishy(1943). Neu (1969) described secondary flows in rivers which he attributed to the coriolis effect. Major objections to this explanation are the observed symmetry of stream meanders and the reversal of secondary flows around sequential channel bends

(Hey and Thorne, 1976). The coriolis effect is negligible near the equator, yet meandering channels occur at all latitudes. Yang (1971) has pointed out that the coriolis force would not effect small scale flows which, nevertheless, meander. Ludin (in Leopold and Wolman 1960) has determined that the coriolis effect on a stream flowing at about 1m/s at a latitude of 60° would only account for a bend of some 13km radius. However, Dinga (1970 in Chitale 1976) has observed a sharper radius of righthand (clockwise) turns in some rivers in the U.S.A., which he attributed to the coriolis effect.

(b) Secondary Flow

The major orthogonal component of streamflow is parallel to the banks. However, there also exists a component perpendicular to this main flow. This is known as secondary flow (and when its pattern is a spiral contained by the river, helical flow) and has long been recognised (eg. Thomson 1876). The magnitude of the two components is dependent on the position in a meander bend, but the polarity of the secondary flow is reversed through sequential bends. Hey and Thorne (1976) identified twin cells of secondary flow in meander bends which exhibited surface convergence at the bend apex and divergence at the crossover point. However, their findings appear inconsistent with the sediment distribution on the river bed. While many authors have recognised the presence of secondary flows in meandering channels and acknowledged its importance in sustaining meanders (eg. Leliavsky 1955, Leopold and Wolman 1960), it remained unclear how these flows originated.

Bagnold (1960) claimed that secondary flow was a result of the meandering condition, but Prus-Chacinski (1954) and Leliavsky (1955) argued that secondary flow was inherent in the flow of water and was the mechanism responsible for meandering. Certainly, flume experiments (apart from those in which water was entered at an angle, eg. Matthes 1941, Friedkin 1945) had developed meandering channels from initially straight ones (Leopold et al 1960, Ackers and Charlton 1970a,b).

Theoretical work by Einstein and Li (1958) and Einstein and Shen (1964) suggested that secondary flows would develop in straight channels. Drawing on Callander's (1969) work on instability, Engelund and Skovgaard (1973) showed that meandering can be expected, given a channel with dune bedforms.

The theoretical work of Yalin (1971, 1972) on turbulence indicated that a horizontal analogue of the vertical eddies responsible for bedforms might be expected. Such turbulence would develop zones of high velocity, spaced regularly downstream against alternate banks. Furthermore, several distinct scales of meandering would be possible. The scale of such meanders would be controlled by width, a relationship long-predicted by field and flume evidence. This suggests that the importance of discharge is in controlling width. It remains unclear, however, how the migratory and ephemeral turbulent eddies invoked by Yalin are related to the relatively invariant form of the river meander (Parker 1976 pers. comm.).

(c) Sediment Type

Flume channels have been observed to produce bars with both lateral symmetry (Leopold and Wolman 1957) and asymmetry (Ackers and Charlton 1970b). It seems that the development of meanders from these forms requires the injection of sediment (Tiffany and Nelson 1939, Ackers and Charlton 1970b). Hakanson (1973) has pointed out that symmetrical bedforms are essentially ephemeral features, because there are two lateral zones of maximum turbulence and minimum bed stability which are either side of the centreline of a straight channel.

Quraishy (1944) claimed that secondary flows were unimportant in the initiation of meanders which occurred as a result of the "... interaction of moving water and sediment..." creating alternate bars. Sheperd and Schumm (1974) found that an initially meandering channel, superimposed on fine, cohesive material, lost its meandering form. The fine material, once eroded, remained in suspension and did not maintain the point bars. Maximum scour occurred on the inner edge of the bends, eventually forming deep pools. Hooke (1975), working in a pre-formed meander bend in a flume, described high shear stress values on convex banks.

He suggested that this was normally expended in transportation of material derived from the upstream concave bank. This could explain the scour observed by Sheperd and Schumm and implies that both transportation and deposition are important in meander propogation and maintenance.

The importance of inherent secondary flows is felt to be exemplified by meanders in supra-glacial streams (Leopold et al 1964, Knighton 1972), ocean currents (Stommel 1965) and small unconstrained flows on glass (Tanner 1960). Some proponents of this school of thought are Prus-chacinski (1954), Leopold and Wolman (1957), Karcz (1971), and Goryki (1973a). The analytical work of Einstein and Engelund supported this view.

Parker (1976), attempting to determine the source of instability leading to the meandering of river channels, found sediment transport was important. He accounted for meanders in supraglacial streams, ocean currents and small unconfined flows respectively through thermal considerations (Parker 1975), the coriollis effect (Stommel 1965) and surface tension (Goryki 1973a,b). The similarity in proportionality between these types was due to the common factors of inertial and gravitational potential, and frictional effects. The actual source of instability was different, and not necessarily important (Parker pers. comm. 1976).

It is not within the scope of this thesis to critically evaluate the material described above. However, it does seem that at present the analytical approach is the most promising with respect to establishing a physical explanation of meandering phenomena. Although meander scale is flow-dependent, it is clear that other factors are of importance. The effect of bedrock on meandering has been recently discussed in the geographical literature.

(vii) Bedrock and Cave Meanders

(a) The effect of Bedrock

Brush (1961) showed that stream slope was controlled by the strength of the country rock and Hack (1957) had suggested that this might be related to the size of particles in the stream. Schumm's (1960) work in the Great Plains showed that width-depth ratio in alluvial channels was inversely proportional to the percent silt-clay in the channel. His observations on the effect of cohesion (1963,1967) have already been noted.

Jefferson (1902) noted that meander size was variable and that especially large meanders were found where the stream cut into bedrock. Bank strength as well as discharge seems to control meander wavelength.

Dury (1964, 1976a) has called attention to small streams which occupy a floodplain in a large valley; underfit streams. Davis (1913) had invoked capture as an explanation of this apparent shrinkage, but Dury recognised the phenomenon to be very widespread and sought a palaeoclimatic explanation. The valley itself was often found to meander and the scale of these meanders was related, using regime equations, to discharges some twenty-five to one hundred times greater than those experienced now. He suggested that rainfalls one and a half to two times as great as at present could account for this scale of runoff, which formed the large valley meanders.

A more subtle disequilibrium may also be recognised, the "Osage" type, in which large alluvial meanders are occupied by an underfit stream in which a sequence of pools and riffles, apparently unrelated to meander form occurs (eg. Dury et al 1972). Remarking on the observed tendency for pools (deeps) and riffles (shallows) to form in straight channels (Leopold Wolman and Miller 1964, Yang 1971, Richards 1976a,b), Dury claimed that a river which has experienced a recent decrease in discharge would have a lowered erosive capacity. It would, therefore, be unable to alter the planform meandering formed by higher flows. Alterations in bedforms would be possible, however, and the river would maintain a characteristic cyclicity in bedform, rather than in planform (Richards 1972, Ferguson 1973a).

Subsurface studies have confirmed that "meandering valleys" are bedrock features which possess cross-sectional and longitudinal profiles comparable to those of alluvial rivers (Dury 1964). However, a "preferred position model" (Palmquist 1975) can account for some of the characteristic asymmetry. If a river is capable of eroding its valley floor by scour during high floods, the tendency for underfits to occupy the outside of valley bends will concentrate erosion at these locations. Dury (1976b), in reply, has pointed out that the age of some valley fills implies no scour has taken place since shrinkage of the original stream.

Dury implied that stream shrinkage was responsible for the aggradation of his valleys. This could stem from other environmental changes, or a relative rise in sea-level, or more probably a result of overloading of streams in response to some disparity between slope and fluvial processes. His thesis demands a pluvial period over large areas of the world from 12,000 BP. for European examples, to 2000 BP. for some recently emerged valleys around the Great Lakes (Hack 1965). It seems unlikely that the general circulation could maintain the extended, global pluvial periods required.

Cogley (1973) proposed that glacial meltwaters would provide discharges of the size demanded by Dury, although Dury (1973a) rejected this explanation as inappropriate for low-latitude examples and unsupported by field evidence for temperate examples. Prus-Chacinski (1973) noted that modern floods often exhibited large scales of meandering and suggested that these infrequent events may be of importance. Dury (1972, 1973b), however, did not accept modern floods as effective erosive agents. The large and frequent discharges characteristic of tidal channels have been related to many oversize meandering valleys by Geyl (1976a,b).

Hack (1965) described active, large scale meanders in sections of river armoured with large boulders or in bedrock. Alluvial reaches of the same rivers displayed more conventionally sized meanders (as well as sections of compound or multiple meanders). Tinkler (1971) described similar situations from Texas (see Table 1.), and Chang and Toebes (1970) found bedrock reaches of some rivers in Indiana had larger wavelengths than contiguous reaches cut in till. Tinkler (1972) and Kennedy (1972) suggested that meanders are characteristic of the discharge most effective in eroding the banks; the greater strength of bedrock requiring more energy for erosion than alluvial meanders. Thus the channel-forming discharges would be greater, and have a longer recurrence interval than those in alluvial rivers.

The underlying question in this debate is whether the higher discharges responsible for the bedrock valley meanders are merely a rare component of a river regime similar to present regimes, or whether they stem from past pluvial climates. It is felt that the evidence is for the former. However, Dury (1972) has pointed out examples of large valley meanders cut in weak material (eg. glacial outwash, alluvium, loess and swamps), and the erosive effect of the infrequent, large discharges is unknown (compare Dury 1973b, and Stevens et al 1975). The overbank stage has been found independent of the channel, which merely constitutes an additional source of roughness to the high stage flow (Smith 1977).

Schumm (1967) found that channels with cohesive banks maintained shorter wavelengths than those in non-cohesive alluvium. This implies that stronger banks do not necessarily demand higher discharges for erosion. He placed emphasis on the type of sediment load; clay and silt were suspended load, sand and gravel saltated load and larger particles comprising the bedload. The flow's ability to transport, rather than erode, material seems to relate more closely to meander scale. Kellerhals et al (1976) pointed out that gravel bed rivers generally exhibit different bed forms from sand bed rivers. This implies that river processes responsible for form are related to sediment size. Winkley (1977 pers. comm.) pointed out that bedload appeared to be important in sustaining meander migration.

Kirkby (1972) drew attention to the distinction between "erosional" meanders and "transportational" meanders (author's terms). The former occur where, once eroded, all sediment is removed, and the latter store sediment in the bed and bars. He observed that meandering and braided streams can be distinguished using roughness criteria, and that the same criteria also separated alluvial and bedrock meanders. He suggested that manifest braiding required an abundance of sediment in the absence of which large meanders would form, although Sheperd and Schumm (1974) found that meander forms were destroyed due to scouring of convex banks in situations where all sediment was entrained. When a small volume of material was deposited, however, erosion of the concave bank was reinitiated, ultimately leading to meandering of the channel.

The relationship of analytical work to bedrock meanders is unclear. If discharge directly influences meander scale, and a larger flow is necessary to erode stronger material, then larger meanders would form. Yalin's (1971, 1972) explanation of meandering depended on width as the critical scaling factor. Width is determined by discharge and bank strength, and bedrock streams tend to exhibit relatively narrow channels which would lead to smaller meanders.

Although Yalin's work appears inappropriate, Parker's instability considerations may be applicable to bedrock meanders. The implications are that sediment transport mechanisms are an important control of meander scale.

(b) Cave Meanders

Limestone and classical aquifers differ, in that the former store a large proportion of their water in fractures (secondary pores), rather than within the primary pores of the rock itself. These fractures are joints, bedding planes and faults. As water moves through the limestone, solutional erosion enlarges the fractures. Generally, a network of tubes (anastomoses, Ewers 1966) is formed. As a tube is enlarged water flow becomes turbulent. This is claimed to result in a faster rate of solution of the limestone, an increased rate of enlargement of the conduit, and the capture of flow from other fissures and small tubes (Wigley 1973).

The cave passage behaves very much like a surface river in draining a catchment. Small fissures contain slower-moving percolation water which is responsible for the maintenance of baseflow (White and Schmidt 1966, Atkinson and Drew 1974, Atkinson 1975).

The form of the cave depends upon the interaction of the hydraulic and structural gradients, and the chemical properties of the limestone and the water. Caves are classified into two end-member types; phreatic, where the entire passage is (or was) water-filled, and vadose, where a stream only flows (or flowed) across the floor.[7] All caves are initially phreatic and inspection of the roof of a vadose passage may reveal phreatic features which have been abandoned by subsequent downcutting of the vadose stream. If the cave developed from a bedding plane, the T-type passage may occur. A phreatic rift may be observed where a joint or fault initiated the cave and a long period of residence in the phreatic state may cause a tube to form. The tube is a result of outward erosion, rather than the predominantly downward erosion of the vadose stream (See Figs. 3,4a).

The initiation of the vadose passage from the phreatic stage may result from some decrease in discharge, or simply from growth of the passage. The recurrence of phreatic events in vadose caves is of age-significance (degree of development) only. It is not equivalent to a bankfull discharge, although an underground floodplain, and consequently a bankfull discharge may exist in some caves

(Jones 1971).

In caves where limestones are massively-bedded, undistorted and gently-dipping, and the hydraulic gradient is concordant with the structure, the hydraulic nature of flowing streams may be manifest. In this situation vadose caves exhibiting apparent meandering may occur. The phreatic cave may also meander, but has remained unstudied, because of the multidimensional nature of the problem and uncertainty concerning the effect of increase in cross-sectional area on meander scale and growth.

Structure can influence stream morphology eg. Strahler (1946) noticed distortion of meander growth due to slaty cleavage. Nutter (1974) observed surface streams following joints, and Campbell (1973) has described development of false meanders forms because of joint effects. Structural control of caves is implicit in most modern speleogenetic speculations (eg. Waltham 1970, Wigley 1972). Cave morphology is controlled by bedding planes and insoluble beds (Waltham 1970, Currens 1975), faults (Gregg 1974) and joints (R. Deike 1969, Weaver 1973). G. H. Deike (1967), and White and White (1974) emphasised the importance of baselevel in controlling cave development and discussed the relative importance of various structural features in different situations and stages of cavern evolution. The interaction is by no means simple.

Some cave meanders show no direct evidence of structural control nor any evident vertical fracturing, and these are possibly "free" meanders. As the cave stream cuts down, earlier meander forms are preserved in the walls above, often presenting a pattern of bewildering complexity. In places a progressive migration of meanders appears to have occurred (generally in a downstream direction; Tratman, Ford pers. comm.). In other locations discrete levels with distinctive meander patterns occur (See Fig 4b), perhaps as a response to changing discharge and erosional processes, or to lithological factors. The difference between these two types remains unstudied.

Davis (1930) and Bretz (1942) remarked on the meandering nature of some cave passages. In their examples, the meanders were apparently inherited from meanders in overlying detritus and were actively increasing in size. The first specific study of cave meanders was by Wheeler (1967), working in Irish caves. The meanders were found to be related to width, but exhibited wide scatter. There was great difficulty in sampling well-formed meanders and the field measurement was most laborious.

Deike (1967) in an important study of Mammoth Cave in Kentucky described meandering (phreatic) tubes and (vadose) canyons. He defined wavelength as twice the bend-spacing in order to establish a large sample. He found an average linear relationship of 5.4 between wavelength and width, but his results showed substantial scatter. This compares with values of 6.6 (Inglis 1949) and 10.9 (Leopold and Wolman 1960) in slightly non-linear relationships. (Table 1). He found that higher wavelength values were produced by canyons than tubes. In one location a large passage exhibited meandering of a similar width-wavelength ratio to a small stream which was incised into its silt floor.

Ongley (1968) attempted to define hydraulic parameters for a short, apparently-meandering cave in Australia. His work is disappointing, for although he attempted an objective analysis of the passage form by discretisation, he drew his conclusions from a wide scatter of results. He found an average wavelength-width ratio of 5.5, although his values ranged from 2.88 to 8.55. He was, however, implicitly aware of the danger of aliasing and tested for this using two different leg-lengths.

Deike and White (1969) drew upon a large number of cave surveys to compare the wavelength-width ratios of meanders from Missouri caves with that from other areas, both in the U.S.A. and abroad. They eliminated caves showing structurally-controlled meanders where "...bends are sharp, the reaches between bends are straight and bends are not evenly spaced...." It appeared that joint-control was important where hydraulic gradient was diagonal to an orthogonal joint set. Using methods of measurement similar to Deike (1967), they found caves in Missouri different to those in other areas. This they attributed to the consistent nature of the Missouri Cave surveys. The results were approximately linear with coefficients of 6.8 for Missouri cave meanders and 8.2 for those of other areas. Once again the values lay within the findings for alluvial meanders.

High (1970) pointed out the danger of using cave surveys in speleomorphological studies. A cave survey is generally made with as few survey legs as possible. This means, for example, that the easiest survey route may be taken where there are multiple levels. Where the amplitude of meander oscillations is comparable to the passage width a single survey leg might truncate a series of meanders. High determined that the ancmalous points Deike and White had plotted for some Irish caves had been produced from such surveys. The error was corrected by using field measurements.

Baker (1973) described cave passages in morphometric terms and found little difference between caves and surface streams. Discharge was related to basin area for example. He compared meanders to width and discharge and claimed his findings were closer to those of Leopold and Wolman (1960) than to Deike and White. A reconsideration of Baker's data, however, indictes that his findings were in fact closer to those of Deike and White. Furthermore, his graphical material was apparently misplotted. Table 1 presents relationships between width and wavelength from both caves and surface rivers.

The author presented a short dissertation on cave meanders in 1973, the data and some of the findings of which are included in this study.

Kuniansky (1974 in abstract) studied several meandering caverns and concluded "...that the hydraulic geometry of cavern passage meanders, extensively modified by vadose waters, tentatively corresponds to the geometric meander parameters for surface streams." It is unclear whether or not the "vadose waters" were considered to be in equilibrium with the meander forms.

In 1970, Hanna and High proposed the application of spectral analysis to underground streams. While working from a survey in an illustrative example, they stressed the necessity for field investigation and suggested a field method for the direct compilation of discretised data. Unfortunately, their data may have been somewhat aliased. They applied a direct Fourier transformation, a process which will produce only a very generalised spectrum, especially if it is not filtered.

It seems that cave meanders are generally of similar form to alluvial meanders. This is not expected in the context of bedrock meanders discussed above. This may be a result of the nature of erosive processes in the cave meander.

Cave streams erode both by solution and abrasion. Newson (1971a,b) claimed that most abrasion occurs at high flows and most solutional removal at lower flows, on average. The abrasive process depends upon a supply of sediment which may come either from surface streams, cave fill or be produced by the breakdown and erosion of the limestone itself (White and White 1968). The high solutional load at low flows is due to the importance of the saturated percolation water component and solution within the cave passage is of variable importance (Smith and Mead 1962). It seems that given solid load, abrasive action might be of most importance. Perhaps electron microscopy of the cave wall material may allow resolution of this problem (eg. Bull 1976). However, Ollier and Tratman (in Tratman 1969) considered solution to be of greater importance on morphological grounds. They pointed out that the selective erosion of limestone beds and the resistance of "weak", but insoluble, chert beds to erosion could not be reconciled with an abrasive environment. They also considered the vertical evolution of meanders (no downstream migration) and the presence of undercut meanders to be due to solutional processes. Finally, they pointed to the ubiquitous scalloping of cave passages as an indication of solutional erosion.

(c) Scallops

Scallops are concave, asymmetrical, dish-shaped indentations varying in length from a few millimetres to several tens of centimetres. They occur in conjugate assemblages on cave walls, roofs and floors (see Fig. 5a and Plate 1). Less common are flutes, regular troughs running transverse to the flow (see Fig. 5a). There has been considerable discussion of their origin and significance.

Davis (1930) identified scalloping on cave walls and recognised it as an erosional form. He drew comparison with sand ripples which are, however, not purely erosional forms like scallops. Bretz (1942) recognised a longitudinal asymmetry in individual markings, in which the upstream edge of each scallop was more steeply inclined than the downstream edge. Bretz suggested that boundary-layer fluid vortices caused and maintained scallops. He considered them to be solutional in origin, because small, weak, insoluble inclusions in limestone stood proud in scallops, and insoluble beds did not display scalloping.

Goodchild (1969) reported that scallop scale has been variously interpreted to be of age significance or representative of erosional type (solutional-small, corrasional-large). It is now accepted that scallop length is related to some flow parameter, both on theoretical grounds (Curl 1966) and field evidence (Glennie 1963 and Eyre 1964, in Goodchild).

Scallops have been observed in surface rivers on both soluble and insoluble rock (Maxson 1940), but Allen (1971) has shown that scalloping is a universal erosive form, and it seems that Maxson's scallops were not the same as those found in caves. In his thorough analysis of erosional forms, Allen considered cave scallops to be solutional in origin, but did not give any specific reason for this view.

Curl (1966, 1974) and Blumberg and Curl (1974) have provided the most rigorous theoretical and experimental analysis of the scallop form. Wigley (1972), using dimensional analysis, showed that scallop form could be expected to depend on the scale of boundary-layer turbulence and the solubility of the limestone. Briefly, the scale of the boundary-layer turbulence is related to momentum of the water flow (given by the Reynolds number) and the solubility is dependent on chemical and fluid properties (the Schmidt number).

However, Blumberg and Curl have shown that the dependence on the Schmidt number is insignificant. They illustrate the behaviour of flow over a scallop (Fig 5b): the outer flow (1) is turbulent stream flow, the laminar boundary layer becomes detached at 2 and remains laminar until 3, where it becomes turbulent. This reattaches at 4, which is the zone of maximum erosion. A part of the flow recirculates into an eddy (5) and the remainder (6) reforms into laminar boundary flow, which itself becomes detached over the next scallop crest downstream.

The erosion rates across a scallop are shown in Fig 5b. The location of maximum erosion at the point of reattachment causes the scallop to propogate downstream at some 60° into the wall.

The distance from detachment to reattachment is determined by a Reynolds number which is characteristic for the flow and depends on the velocity of the main flow as manifest in the boundary shear stress. This is also affected to some extent by the cross-sectional area of the channel. By using roughness criteria characteristic of scallops, it is possible given conduit diameter (in the case of a tube) or hydraulic radius, (for a vadose channel), and scallop length to calculate the Reynolds number for the boundary layer flow. The relationship is shown in Fig 6. From this Reynolds number a stream flow velocity can be calculated knowing that:

$$V=(Ru)/(Lr) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where:

R is the calculated Reynolds number

r is water density

u is water viscosity

L is the Sauter mean scallop length

ie. $L= Li^3/ Li^2$

and Li are scallop measurements.

Goodchild and Ford (1971) observed that scallop length varied significantly between beds of limestone at one location. Wigley (1972) had suggested that this reflected differences in solubility (or the Schmidt number). Blumberg and Curl did not accept this and, furthermore, Goodchild and Ford found that the chemical composition of the rock was of no significance, although thin sectioning had suggested that roughness might be important. Allen (1971) had emphasised the importance of surface defects in initiating scalloping and Curl suggested that smaller scallops might be a response to defects in the limestone, or possibly due to interaction between scallops. The Sauter mean is designed to remove the bias caused by smaller scallops in a sampling scheme.

Curl (1974) pointed out that there are several potential sources of error in applying his analysis to caves. His graph (Fig. 6) is only accurate to plus or minus ten percent. The flow should be turbulent and in a fairly large sized channel without any major bends, which will create variations in the boundary shear stress, as will any marked convergence or divergence of the walls. The calculations are based on time-invariant velocity, but he suggested that scallops are influenced more by higher velocities.

Curl also noted that intense fracturing and insoluble inclusions would upset the regularity of turbulence, and that heavy bed load or clay deposition would preclude scallop formation.

Scallops in caves are probably solutional features, but like meanders, the scallop is a finite feature produced by a variable flow. Possibly the population of scallop sizes in space is representative of the population of velocities in time, but this is unlikely. Curl (pers. comm.), referring to varying flow, has said: "This opens the can of worms concerning non-constant flow and what kind of scallops form and what are the properties of 'non-equilibrium' scallops. I have no ideas to answer either question!"

The erosional processes of some caves seem to be manifest in scallops and this suggests that some form of "dominant" discharge occurs. In vadose streams scallops are observed to decrease in length above the stream. This is to be expected, for larger discharges are usually associated with larger velocities. Yet it remains unclear how, as the stream downcuts, the scallops change their size, especially as the width of many passages remains constant with height above the stream.

Footnotes:

1. A series is stationary when its properties are invariant with distance from the origin.
2. Aliasing is a sampling problem which occurs when the sampling interval is greater than half the highest frequency of a series. This frequency is not resolved and an artificial periodicity may be produced
3. The limited definition of independence accepted here is:

   Covariance(A|B)=0
4. A sample converges as its properties more closely approximate those of its parent population. This is distinct from the problem of closure which results from constraining variables to a fixed ratio. (See Davis 1973)
5. Other authors (eg. Yevjevich 1972, 1974) observe no distinction between the terms random (or probabalistic) and stochastic.

6. The Nyquist frequency is the minimum frequency resolvable for a series. It is the reciprocal of twice the initial sampling distance, or leg length.
7. Ford (1974) gives a fuller descriptive classification of cave types ranging from vadose to phreatic.

CHAPTER III

METHODOLOGY

(i) Field Areas

(a) County Clare

One of the major glaciated karst areas of Europe lies in North West County Clare (see Fig. 7). The caves have been documented in Tratman (1969) and only brief reference is made to them here.

The Carboniferous Limestones are around 450m thick, and massively bedded. The cavernous component is finely crystalline, marine limestone, with occasional chert bands and, rarely, thin beds of shale. There is no recorded faulting in the area, although there is a dominant set of north-south joints, complemented by a subsidiary, orthogonal set. There is no mineralisation in the locality except for calcite fill in some of the main joints. The dip is 2° south-west on average, but varies somewhat over the area with noticeable effects on the speleomorphology.

Shale unconformably overlies the limestone, but is reduced to hilltop outliers in the area in question and is entirely absent farther north. The shale is important as an impermeable catchment feeding water onto the limestone and into the caves.

There has been extensive glaciation, generally lowering valleys and in places depositing thick tills. The area is the southern part of the "Burren", a region characterised by extensive areas of glacially eroded limestone pavement.

The major caves are all located adjacent to the present shale margins, which were defined largely by glaciation. This, plus the absence of any evidence of multicyclic evolution, suggests a postglacial origin for the caves.

The model of development shown in figure 3 holds true for most of the caves of the area, although a roof tube has been observed above Shaft Gallery in Poulnagollum. The caves are generally terminated in low, wide bedding planes, for the canyons are produced only in upper reaches, perhaps a response to hydrochemical factors. The dip and hydraulic gradient are approximately concordant and many caves form an integral part of dendritic drainage networks, although the regional resurgences appear to be graded to a lower sea level and are now submarine. Many caves exhibit meandering sections, although in some (eg. Faunarooska) this is dictated by structure.

The meandering reaches selected for study were considered to be free from such control, although a major joint may have been important in the initiation of some caves (eg. Cullaun I, Fig. 3b). The reaches are all active vadose passages carrying from around two to ten litres per second during the period of study. While some caves have clean bedrock floors, all are known to carry some material up to 10cm in diameter during times of flood. Chemical analysis of streams flowing through these caves has suggested that they are actively corroding (Smith and Nicholson in Tratman 1969), but this may only be a result of the addition of concentrated percolation waters. All passages exhibit scallops of varying scale and prominence.

Care was taken that no major tributaries joined the passages, so that constant conditions in both time and space may be assumed, although perhaps only imperfectly realised.

Two sites were selected in Poulnagollum (E7 of Tratman); one in Shaft Gallery (here called "Shaft") and an adjoining reach in the upper Mainstream ("Mains"). Cullaun I (C1, "Culla") was a less mature cave of lower sinuosity. St Catherines (D5, "Caths") is a tributary to the extensive Doolin system, as is Polldonough (B7, "Polld") to the Collagh River Cave. Detailed descriptions, including surveys, may be consulted in Tratman (1969).

(b) Gardners Gut, Waitomo

The 11.4km surveyed of Gardners Gut ("Gardners") make it the longest known cave in New Zealand (Fig. 9). It lies in the Waitomo Basin, a major karst area, in the King Country of North Island (Fig. 8). The area has not been extensively studied, and most of the following is tentatively based on field observation, although the geological descriptions and some geomorphological effects of the geology are drawn from Kermode (1975 and 1976, pers. comm.).

The main limestone is the Otorohanga Formation (40-60m thick), which is overlain by the Ruakuri (3m) and Tumutumu (18m) members. Upper Gardners cuts through the Tumutumu and Ruakuri and the major part of the cave is contained in the main Otorohanga Limestone. The Tumutumu and Otorohanga are similar: coarse, spary biocalcerenites with conspicuous, flaggy "beds" some 3-20cm thick, which mask the uneven bedding that actually exerts more structural control over the caves. The "residual beds" (Kermode 1975) between the flags contain somewhat less calcite than the limestone itself, and, while being preferentially eroded in surface outcrops (Plate 3), are prominent in underground locations (Plates 2,5). The relative solubility of the two elements is unknown, but the residual beds are certainly mechanically weak. The limestone weathers underground to produce abundant sandy sediment, which is supplemented by material, largely of volcanic origin, introduced through the swallets.

The Ruakuri is a glauconitic, coarse biocalcerenite and, being less flaggy, is more obviously bedded than the Otorohanga. It also appears to erode more by corrosion and supports obvious scallops and flutes (Plate 1). Its importance in controlling speleological development has been pointed out (Kermode 1976 pers. comm.), but remains unassesssed. Swallet streams generally enter the cave in steep drops which span the thickness of the Tumutumu, until levelling-off occurs across the Ruakuri, which Kermode has described as an aquiclude.

The area has been influenced in several respects by the regional tectonic instability. Geologically this has produced a great deal of faulting and fracturing. Nevertheless, the limestone of Gardners is more or less horizontal, but exhibits prominent north-south fractures, which have controlled the form of the cave. A supplementary set of fractures runs north-east, south-west and has also been of importance. The overall planform of the cave is controlled by these two fracture sets (see Fig. 9), and in some reaches fracturing has provided the means of stream cut-off.

The Tumutumu is overlain by siltstones some 100m thick, which provide catchments for streams flowing into the limestone. The gradual downcutting of streams onto the Tumutumu and their subsequent engulfment has resulted in sequential swallet retreat, with progressive stream capture and passage abandonment (Fig. 10). This process has been somewhat complicated by the periodic deposition of volcanic ash over large areas. This produced extreme sediment loads, which may have choked some cave passages.

The caves of the region appear to be multicyclic, exhibiting up to three distict levels (Williams 1975 pers. comm.). This may be a response to changing base-level, or perhaps to downstream changes. The upper reaches of Gardners show two major levels; the higher of which is represented by

Artesian Hill, Boneyard-Apricot Pie, and the Peter Lambert levels, and the lower by the present streamway. The effect has been complicated by swallet retreat and capture, but generally it is assumed that passages more closely graded to the present streamway have been more recently abandoned.

These factors have been combined to produce a possible sequence of events leading up to the present (Fig 10). The main reach studied runs from the point where the Hole "L" stream joins the main streamway down to Waterfall Passage. The evidence suggests that this reach of streamway has not been greatly affected in its development over time by changes in discharge, baselevel, or catchment area.

A view of the surface above the main reach is shown in Plate 4. Water has cut through the (permeable?) siltstone cap and produced a mass of closed depressions; a "cockpit" landscape. These depressions are manifest underground as avens (see Fig. 9) or dome pits. A large number of avens are found beneath the cockpit surface and many (mostly unsurveyed) exist above the meandering passage, or are linked to it by short, high-level passages, some of which are now calcite blocked, while others are actively eroding. A rather large inlet of this type is shown in plate 2. Brucker et al (1972) considered these features to be a result of percolation of small volumes of aggressive water from the surface through to the cave passages. This appears to be true in Gardners also, for there is no evidence that the avens have ever carried large volumes of water. There are certainly no explorable passages at the head of those which were ascended.

It is therefore considered that the main reach, which runs beneath the siltstone cap, has been hydrologically stationary throughout its recent history. This is not true for the short lengths of passage surveyed in the Ruakuri between Helms Deep and the confluence with the Cleft of the Orcs stream, which are generally narrow slots cut into the floor of larger passages. This was probably a response to the capture of Helms Deep headwaters (see Fig. 10).

Structurally, the main reach is not stationary, for it does not follow a single fracture for its entire length. This resulted in sudivision into the segments shown in figure 9. The long reach ("Hole-L", for Long) was initiated largely from a single fracture, or set of fractures (Fig. 4, Plate 8), although in some places this is less straightforward. Fracturing does become more common in the lower reaches of Hole-L (Fig. 11). "Hole-E" (for Extention) follows a south-west, north-east fracture, and has developed from a complex, joint-controlled passage. It is characterised by breakdown which is produced by the interaction of the two joint sets with the meandering stream passage. An example of breakdown due to undercutting of a joint is shown in plates 6 and 7. A lower series, "Hole-J" (for Joint-controlled) was surveyed because it appeared to be partially joint controlled.

The two karst areas may both be regarded as humid and temperate, although other environmental differences are likely, for example their hydrochemistry is probably as different as their geology, but these factors are not considered here. The Irish caves appear more straightforward in their development and are probably much younger (with the doubtful exception of Poulnagollum). They have also been less structurally influenced in their initiation, and meanders have often developed only 1m below the initial bedding-plane phreas. Gardners' early form was largely dictated by structure and only in the later stages have hydraulic factors been dominant, giving rise to such forms as the meanders.

While the overall topological properties of the systems are similar, major differences in the actual pattern of the caves are obvious.

(ii) Field Techniques

The technique of surveying meander series as an equal-interval, directional series was evolved experimentally, as there was no accepted field technique available. A compass was mounted on an aluminum pole which was attached by nylon cord to a similar pole. The pole was kept vertical by the observation of the floating card within the sealed compass unit. No backbearings were taken.

The channel width was measured to the nearest centimetre and the compass positioned in the channel centre.

The compass was sighted onto the second pole which was similarly placed in the channel centre, unit distance downstream and the bearing was read to the nearest whole degree. A record of bearing and channel width was made on waterproof paper.

The selection of the unit distance was initially subjective. A distance was chosen which would appear to maintain a reasonable change in bearing between stations, yet would adequately describe the bends and not intersect the passage walls. It was also important that the measurements should cover a reasonable length of channel in the available time. The leg length defined the Nyquist frequency for the series and it was desirable that this was higher than any naturally occurring frequency. If this were so, then the spectrum for a series would contain some estimate of variable field measurement error as the variance at the Nyquist frequency. A distance of 1.5m was selected and this has been used consistently in order to allow comparisons between different locations.

Subsequent error analysis showed that the overall compass error (2°) is equivalent to an error of 5.25cm in defining the channel centre. The width measurements were therefore not as critical as the orientation. Width measurements were found extremely time-consuming, therefore in order to realize a longer series, widths were not recorded from Gardners, where a series of 380 data points was obtained (but later subdivided on structural grounds).

This ommission was much regretted in subsequent analysis.

The question of meander evolution was tackled by a survey some 2m above the main stream section in Gardners and was called Hole-H (for High-level). The survey proved difficult, but less so than an attempt to reconstruct earlier meander form from successive cross-sections along the stream channel. Distance above the present stream level was assumed to be representative of time, although this remains an assumption, because it is unlikely that the rate of stream downcutting has been constant along all its length. It was hoped to survey the initial fracture from which the passage had developed, but this was not possible because of "ethical" and practical difficulties (Plate 8). The eventual Hole-H survey was assumed to be only accurate to plus or minus 5°, but this was acceptable in the circumstances. An attempt was made to close the survey to the present stream-level by locating, to the nearest stream-level station, the points where the high-level series crossed over the low.

Joint surveys were made to the nearest five degrees in all caves, but it gradually became clear that the joint survey as a frequency distribution was less important than the location of the individual joints in the stream. This culminated in the recording of the approximate location of every obvious joint in the Hole-L and Hole-E series.

Other features such as inlets, breakdown and bifurcations were recorded. The latter forms proved intruiging as they were not observed in the contemporary stream, but only as fossil features in the walls. There was no evidence of structural effects, but it seems intuitively unlikely that they represent a bedrock analogue of braiding, unless the form is inherited from alluvial bedforms. The stream in Gardners appeared to have recently reached a more fractured bed of limestone, and in several locations cutoffs have been initiated.

Gradient was measured from the hydrostatic head, using clear plastic tubing. This was inserted into the stream and the level of water in the tube measured five or ten metres downstream (see Pierre 1970). Gradient was not measured in Gardners, partly because of the difficulties encountered at cutoffs and because the channel was on a sediment bed throughout and grading would possibly have concealed any structural effects.

For Shaft, Culla and Caths a scallop sample was taken at 5m intervals. The longest, longitudinal axis was measured from three scallops on either side of the stream, and at various levels. Scallops were also measured from a small reach of channel below Helms Deep, but were sampled every 1m. There was some difficulty in obtaining a sample without subjective judgement. The scallops were not uniformly developed and the more prominent ones were generally sampled on the tenuous assumption that they were in a finer state of equilibrium with the flow. Ultimately the small, but extensive sample was deemed unreliable, and the measurements from each level above the stream were lumped. The appropriateness of the sauter mean, given the subjective sampling is uncertain. The main reach of Gardners did not support scallops, probably because of the sandy nature of the limestone, the heavy sediment load, and also the interaction of the residual seams with the flow.

(iii) Data Analysis

The main series produced from the field work were; Bearing (referrred to as 'B'), Width (W), and joint patterns (J). Gradient and scallop data were obtained from some locations. The series were transformed in order to facilitate determination of their underlying properties.

The analysis may be philosophically divided into two overall approaches: firstly the series may be "lumped" and the series properties, or average behaviour considered. Alternatively the data may be broken into individual bends (half wavelengths) whose properties may be measured, and then either studied as a series or a distribution. This constitutes a process of discretisation, only the sampling interval is defined by form, rather than by a unit of distance. Each method has its advantages and they are not mutually exclusive. The possibility of structural control tends to render the latter method more appealing, especially where major controlling factors are known to be non-stationary along the stream.

(a) Data Transformation

The mean of each series was calculated as the mean vector, as is appropriate for directional data (Mardia 1972), and the deviations of the bearings from this mean were calculated (the D-series). This transformation has no effect on the higher moments of the series. The first and second differences were also calculated to produce the "dB" and "ddB" series respectively. The D and dB transformations had to be applied with care, because of the 0-360° transition. Therefore, when a series runs from below 360° across this boundary, its numerical values drop. The difficulty was accommodated by assuming that the transition was temporary, and that bearings less than 0° and greater than 360° could be occasionally encountered. The problem was not great, however, due to the fortuitous southward orientation of the cave streams.

The radius of curvature was calculated as the half leg length, divided by the sine of half the change in bearing (see sperare:beth 1974, page 95, for proof). The usefulness of radius of curvature is largely conceptual and is limited by its negative exponential relationship to change in bearing. This means that when a small change in bearing occurs, a one degree difference will produce a large difference in radius of curvature. Therefore, the measured bearing error (which is proportionally greater at small changes in bearing) is magnified considerably by the transformation.

The bearings and leg length were converted to Cartesian co-ordinates, and stream-level plans drawn up using the "McMaster Cave Plot Program" written by M. F. Goodchild and J. Coward. The high-level data from Hole-L (called Hole-H) was modified to close with the fixes obtained on the lower series. Corrections were made to both length and bearing between individual points of closure. The new series was called Hole-N (for New).

The Sauter means were calculated for the scallop samples, which as noted above, were lumped for each series. The validity of lumping was supported by the lack of any significant correlation between scallop length and gradient, or between the difference across the channel and change in direction. The recognised weakness of the data prevented the application of multivariate methods.

(b) Simple Statistical Moments

The mean vector strength (R) was calculated along with the mean vector direction. One minus R is the circular variance, and it varies from zero when bearings are constant, to one when the data is uniformly scattered around the circle. Calculations of sinuosity were made over the full length of each series, which always gives a higher value than the average sinuosity of a series of single bends, because of the irregularity of the bend orientation. The overall sinuosity was compared to the average from the individual bend series. The sinuosity from each series was compared to that calculated from the variance (Ferguson 1977) and to the reciprocal vector strength. When the overall sinuosity was greater than 1.5, the non-dimensional degree of wiggliness was also calculated (Ghosh and Scheidegger 1971).

The bearing data and their transformations were lumped into frequency histograms, and the form of the distribution noted. The dB series was tested for normality as this appeared to be a characteristic of alluvial meanders (eg. Thakur and Scheidegger 1968). The observed and expected frequencies were compared using the Kolmogorov-Smirnov (KS) one-sample test (Siegel 1956 and Mitchell 1971). The KS test is more useful than the Chi-square test in situations where differences other than in the mean are anticipated in the data. The test acts conservatively when certain of its assumptions are not fully realized, which allows one to maintain some measure of confidence in its results.

Although Mardia (1972) has described the statistical tests appropriate for circular data, these were not understood sufficiently to rationalize their application. The dB series is in some respects suitable for treatment as linear data.

The kurtosis of the dB distribution was calculated, because Chang and Toebes (1970) had found that the distribution of dB for bedrock meanders was more leptokurtic (peaked) than that for meanders in till. Although this implies meanders of somewhat larger wavelength, it may equally represent lower amplitude. The physical interpretation is therefore ambiguous.

(c) Structural Control.

One of the major considerations of the present thesis is whether the meanders in caves are free meanders. The passage's response to water flow is accepted, but how much is the meander form constrained by structural effects? There is no absolute geometry which can be expected for the cave meander; the wavelength of meanders in bedrock has been noted as an area of debate. It is necessary, therefore, to establish properties of the meanders from the series themselves, so that any irregularity may be determined.

The distributions were studied to see if any effects were obvious, such as a preferred direction away from the mean. This might occur if the dip were not parallel to the fracture orientation. The Hole-J distribution was also used as a control.

The joint surveys were lumped into frequency distributions and compared to the distribution of directions for each series. No rationale for a statistical comparison was available. This is partly because of the axial (0-180°) nature of joints compared to the full circle distribution of direction, although this may be combatted by halving or doubling of the appropriate distribution. The joint sample is a discrete distribution, sampled whenever it is available. The bearings are a series which is serially correlated; a continuous distribution discretised into a finite data set. The two elements are not independent in their sampling. In order to test for any similarity, the two sets must be independent and random. The joint sample was taken from joints intersected by the passage walls. If a particular joint were directing the passage, it would not intersect the passage and joints would be sampled more frequently in proportion to the angle they make with the controlling joint. Corrections are available for this sampling error, (Terzaghi 1965), but they are difficult to apply in a meandering cave. The joints are, furthermore, not uniformly distributed along the channel, and to lump them into a histogram implies that the distribution of joints is acting on the passage at all points (or is stationary). The joint is effective only where it is intersected.

No statistical tests between joint and passage distribution were made. It is likely that any differentiation of the samples would have been unreliable. The inability of earlier attempts to relate passage and joint distributions, even in manifestly joint-controlled passages, has been referred to above, and this makes the likelihood of establishing any such relationship in a "wandering" passage very small. Only a qualitative comparison was therefore made.

Alternative methodology was developed to tackle this problem. An approximation to an expected and observed distribution is possible to construct from a long series, if the precise locations of the joints are known. The direction and change in direction series may be divided into those elements occuring where joints intersect the channel and where they do not. If the joint control is limited in this manner (which is unlikely), it is possible that the effect will be manifest. Large changes in direction were noted to occur where a joint bisected a meander bend in some Irish caves.

Jointing is assumed to be an homogeneous population, thus each observation is assigned an equal weight. This is inappropriate, because different families of joints exist in space, and a single joint varies along its length. Not only do joint properties vary, but the interaction of the water with the joint depends on several external controls; the hydraulic gradient, the stream velocity and volume, and aggressiveness, for example. Therefore, it is impossible to predict the extent to which a joint may control a particular cave passage. Even if the jointing itself was homogeneous, it is not known if a stream would follow the joint precisely. The amount a passage direction may depart from a "controlling" joint is unknown.

Ongley's (1968) test of the frequency distribution of number of bearings per degree class against a Poisson distribution was made. The distribution was compared to the Poisson distribution using the KS test. There are conceptual difficulties in the application of this test, however. It assumes a particular joint or set of joints would control the passage in an absolute manner. It is unlikely that passages would conform to jointing in this way, because of the inhomogeneity of the joints and the momentum of flowing water. The test was modified to 5° intervals to compensate for this, although the test is less powerful as a result. It is an assumption that the Poisson distribution is an appropriate descriptor of free meander form.

A less obvious form of structural effect was observed in Clare. Shelving in cave walls consists of sharp, horizontal ledges which appear to be produced by the preferential erosion of invisible discontinuities in the limestone (Tratman 1969). A preliminary study of this feature suggested that it was directionally dependent, because it was more frequently observed when the passage was running in a particular direction. Although no conclusive evidence has been gathered, the possibility of directional control was examined by plotting width against direction. A regression is not appropriate as it would test only for a linear relationship between the two variables, whereas the relationship is probably more complex. Also, the numerical "values" of direction are not magnitudes as are passage widths, which means the directions are merely a type of nominal data in this case.

Structural control must, therefore be a field observable, and voluminous data has no power for such problems when it is considered out of context. The effect of structural control is possibly manifest in other aspects of the analysis, for example the symmetry of left and right bends and the downstream persistence of series properties.

(d) Meander Properties

1. Series

There are some meander properties which might be expected from studies of the alluvial case. The ratio of radius of curvature to width has been observed to frequently lie around two to three. This was tested in the Irish data. Previous consideration of this relationship does not recognise the domain problems encountered in calculating radius of curvature for small changes in direction. If a change in direction of 1° is recorded, the expected width of passage for the Irish caves would lie between 70m and 430m, quite apart from the infinite width associated with a straight channel! The relationship is obviously not a straightforward one, but was investigated as a possible meander characteristic.

Surkan and Kan (1969) reported a directional dependence of the standard deviation of change in direction. This possibility was tested for by dividing the deviation from the mean direction into 10° intervals and measuring the standard deviation of the changes in bearing associated with each class. The class sizes at the extremes were often very small and an attempt was made to compensate for this by dividing the standard deviation by the class frequencies, the square root of class frequencies, or the mean change in direction for the sample class.

2. Individual Bends

An individual bend was defined as a single run of sign of curvature (change in direction) following Ongley (1968). Single membered runs were ignored as unresolvable perterbations of the passage. The individual bend is equivalent to half the wavelength of a meander, but allows resolution of relatively "short-lived" behaviour of the planform. The parameters of each bend calculated were: the mean deviation (in tables referred to as MeandB), the standard deviation of change in direction (SdevdB), the bend spacing (Wavele, half normal wavelength), sinuosity (Sinuos), stream length or arclength (Arclen), the orientation of the line defining wavelength (Orient), and, in addition, for the Irish data: mean width (Meanwi), and standard deviation of width (Sdevwi).

A comparison was made between the average, standard deviation and coefficient of variation of each parameter for each series to determine if any systematic variation existed. The differences between right and left bends in each cave were studied for each parameter to see if there was any consistency within regions, as a response to the coriolis parameter, for example.

The tendency for high or low values of parameters to occur together, or the possession of the Markov property, was investigated by autocorrelation of the parameter series. While series of elements less than a single bend have been fairly widely studied, there does not appear to have been any consideration of the manner in which bends follow one another. A lag of twenty percent was used, which would have caused some loss of information.

Correlation matrices were calculated for each cave to determine if the relationships between the parameters could be confirmed, in both a general and specific sense.

The frequency distributions of values for mean change in direction, wavelength, arclength and orientation were also calculated to investigate the nature of the variability of the samples. The mechanism of bend definition limits the resolution somewhat, but it was felt to be appropriate in order to facilitate comparison with earlier studies.

(e) Stationarity

Most series sampled were not long enough to render a comparison of its properties for different segments more than a trivial exercise. The greater length of Hole-L, plus Hole-E permitted this, however. A running sample of 100 points was taken along the length of the series jumping by ten points each time. Estimates of mean and vector strength were taken on each sample. The fluctuations in the sample were probably as much a response to the position of the series with respect to individul bends as an indication of stationarity. Linear trend in the raw data was tested by simple linear regression of order against the width and the transformed bearings (ie. deviation from mean direction, curvature and change in curvature).

Linear trend in the meander bend properties was also tested for by linear regression. Hole-L was divided into two parts and spectra calculated for each.

(f) Markov Properties

The information contained in the transition probability matrix (the TPM) was studied as a descriptor of the overall properties of the series and their transformations (see Fig. 2). The series used were made up of runs of positive and negative numbers which were taken as the characteristic states. The series studied were; the deviation from the mean bearing (D), the change in bearing (dB), and the change in change in bearing (ddB). Before the matrices were composed the series were truncated so that a number of whole cycles, or wavelengths, were considered. This was found to have significant effects on the matrix values. Zero transition values were set at plus or minus one degree, following the sign of the preceeding value.

The TPM was tested for symmetry across its correlation diagonal by comparing the actual values with those expected for a state of perfect symmetry using a Chi^2 test

The change in value of correlation and anticorrelation diagonals in transformation carries information on the nature of the series periodicity. In a regular wave in which each cycle does not depart from the mean direction, there is no change in the matrices of the deviation from mean and curvature matrices. The amount of change in the matrix reflects the extent to which oscillations occur away from the mean direction. In a wave where curvature is a minimum at the crossover points and a maximum at the apices, the change in curvature has twice the frequency of curvature. There is no such periodicity in a circular wave. The changes in values in the curvature versus change in curvature transition matrix, therefore, show the regularity of changing curvature. Changes in the correlation diagonal contain information on the conservatism of states, while the anticorrelation diagonal demonstrates the number of changes in state. The calculation of the above for the transition matrix gives the absolute number of each state gained or lost. The transition proportion matrix summarises the proportion of each state, and is therefore more useful in comparisons.

The utility of the matrix as a descriptor of series properties was dependent on the strength of the Markov property or memory. This was tested using a method outlined in Harbaugh and Bonham-Carter (1970) in which a Chi^2 statistic is calculated from the TPM and transition matrix.

The extent of the Markov property was also calculated by seeking the limiting matrix. The limiting matrix is one in which the rows are identical (In other words, the following state is independent of the preceding state and is merely controlled by the relative frequency of the particular states in the series). It is calculated by sucessive multiplications of the original TPM, which is analogous to movement along the series according to the Markov model. When the limiting matrix is arrived at, the number of iterations performed defines the range over which independence is acheived. The limiting matrix shows whether one state is preferred over another in a series, whereas the symmetry test compares the relative number of conservative transitions (ie. where sign does not change).

The presence of asymmetry may be variously interpreted, depending on the initial transformation to which it is applied. The direction (D) series shows the preference of directions to one side of the mean direction, which might be of some structural significance, providing the series maintains a general tendency towards the mean direction along its length. The curvature (dB) series compares the radius of bends to the right and left, and the change in curvature (ddB) series matrix may describe variations in the rate of increase or decrease of curvature along the series, which indicates whether the series is symmetrical in terms of stream direction (See Table 28). The ddB matrix is more difficult to explain in terms of bend properties, as this would require the dB runs to be first order curves with only one inflection. The dB and ddB matrices probably are related as much to hydraulic as to structural factors. Any clear asymmetry was compared with asymmetry observed in the average right and left hand bend parameters.

The TPM can only be interpreted on the assumption that the series is stationary. The descriptions are of average bend properties, and in short series the matrix is especially sensitive to "anomalous" bends. This was checked by testing individual bend properties for both right and left bends. A Chi² test for stationarity, documented in Harbaugh and Bonham Carter (1970) was applied to the series which was divided into three approximately equal parts.

The Markov chain analysis was not developed, partly beacause of problems of stationarity and partly because the higher order transformations (eg. the fundamental and mean-first-passage-time matrices, Collins 1975) were difficult to interpret in both abstract and physical terms.

(g) Spectral Analysis

Spectral analyses were performed using a "BIOMED" package; "BMD:02T, Autocovariance and Power Spectral Analysis" (Dixon 1968). The program offered several optional transformations of the data, although only detrending was applied. The number of lags chosed was 15% of the series length, as this appeared to produce an acceptable level of resolution without untoward data-loss, nor production of a "jittery" spectrum. The curvature transformation was used in calculating all spectra, because the series contains oscillations characteristically fluctuating about crossover points.

The frequency bands of the spectra were converted to wavelengths, and compared to the population of individual bend spacings for the same reach. The spectra were difficult to interpret for a number of reasons. The band width was rather large (ie. resolution was not good) even with the largest number of lags (30 for Hole-L). The spectral estimates (as frequencies) do not convert to wavelength linearly, and the wavelength equivalent of the spectral estimates are highly attenuated at large wavelengths (low frequencies, where a single band might cover 10m) and finely resolved at smaller wavelengths (eg. 3-5m wavelength).

No useful significance tests were available for single spectra, but evolutionary changes were inspected by comparing the spectra from Hole-H and Hole-L (the upper and lower levels along one reach. The Hole-L series was tested for stationarity by comparing spectra from two halves of the series.

(h) Scallops

The scallops were reduced to the Sauter mean (see Chapter 2) and Curl's (1974) methodology applied. Widths were measured at the time of scallop sampling in Culla and Caths. The channel width over mean scallop length was calculated and the characteristic "scallop Reynolds number" calculated from figure 6. The channel was assumed to be a "parallel walled conduit", but this may have caused an underestimate of Reynolds number especially for the stream-level samples, because flow depth is not great and frictional retardation of the bed is not considered in the method. The characteristic flow velocity was calculated from equation (1) in Chapter 2 (section vii, c).

A reversal of the Manning equation allows the hydraulic radius (R) to be calculated. from the slope (S), velocity (V) and a characteristic roughness value (n) as:

$$R=((Vn)/S^{05})^{15}\ldots\ldots\ldots\ldots(2)$$

The value for n was taken as 0.025 and was derived from White and White (1970) and field estimates made in scalloped passages in Northwest Yorkshire, England. n is a dimensional constant and all length units must be expressed in metres when the Manning equation is used.

The hydraulic radius is related to width (w) and depth (d) by:

$$R=(dw)/2d+w\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

This equation can be reversed to allow calculation of depth thus:

$$d=wR/(w-2R)\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Knowing mean width from measurements, velocity from scallops, and having calculated the depth value, an approximate estimate of "scallop-forming discharge" can be calculated.

CHAPTER IV

RESULTS

(i) Data

The cave passage plans were reconstructed from the bearing data and automatically drawn using the McMaster Cave Plot program. The plots were reduced to suitable scales for single page presentation (see Figs. 11-19). The high-level survey above the Hole-L stream (Hole-H) was found, as expected, to diverge from the lower survey because of survey error. In analysis, the raw high level data were used where constant unit sampling interval was important, for example in frequency distributions. However, in order to have some control over meander form, the high-level series (Hole-H) was closed onto the Hole-L series at recorded crossing points. This produced Hole-N, a new series, theoretically more accurately portraying the high-level planform, but not composed of equal sampling intervals.

Although width data were recorded from the Irish caves (Table 2), resolution of width at the chosen scales on the plans would have been inadequate. The average width is shown on each diagram. The term "plan" is loosely applied since the surveys are "stream bed" plans with no corrections for slope. The effect is negligible, however, since the highest gradient recorded was 0.031 for Caths (Table 2), which produces a difference of only .07m between stream length and horizontal distance.

The process of discretisation averages passage form in a nonuniform manner. This is of little significance where curvature is relatively low, but in Hole-J for example, some very sharp corners exist and these have been smoothed by the survey.

Additional material is marked on the plans where appropriate. Hole-L and Hole-E have both joint location and orientation information. The trend of Hole-L is seen to correspond to the major fracture axis, and the departure of Hole-E from this course may be attributed to the presence of more complex fracturing. The straight reaches of Hole-J run along fractures. Few joint locations were recorded for the Irish caves, and an apparent association of sharper meander apices with intersecting joints was not borne out by mapping of joint locations in the Gardners' meanders.

The stream cutoffs in Gardners were all a result of joints capturing the stream. Although cutoffs formed by migration and intersection of meanders may be observed in caves, they are comparatively rare. Most cutoffs recorded were recent and many were yet unable to carry even moderate discharges. This cutoff-forming process is characteristic of cave meanders, and is one form-modifying mechanism likely to create differences between cave and surface meanders.

The inlet streams noted were small (discharge normally less than 1l/s), and unlikely to greatly alter the discharge of the main stream. The bedrock collapse (breakdown) was usually a result of the stream undercutting fractured blocks. This was especially noted in the Hole-E reach, where intersecting fracture sets further undermine the stability of the passage walls.

The raw bearings were transformed (as described above) into curvature and change in curvature series. The voluminous nature of these data, along with the individual bend characteristics, precluded inclusion in the text. However, Appendix A gives details on their availability.

(ii) Distributions

The frequency distributions of direction, curvature and change in curvature are presented in tables three, four and five respectively.

The direction data are distributed in a complex, but approximately normal form (except for Hole-E). Bluck (1971) showed regular waveforms to have bimodally distributed direction (eg. Mains, Shaft, Hole-E), but Ferguson (1977) considered the irregularity of natural rivers to render a normal distribution of direction more likely. The exact shape of the distribution depends on the stage of development of meanders, as well as on the nature of external constraints acting upon them. Here, the bimodal distribution appears to represent the more fully developed meanders, where a relatively high amplitude-wavelength ratio exists. The peak at 50-59° for Hole-L might have been considered a structurally affected element, except that no fractures were recorded with this bearing. The Hole-L peak at 90-139°, however, does appear to correspond to a fracture axis. The fractures do not always coincide with the mean direction for the passage (eg. compare Mains which does coincide, with Shaft which does not). This can be interpreted as a difference between the structural axis and the direction of the pressure gradient (or gravity gradient in vadose conditions), and illustrates the interaction of hydraulic and structural factors at different scales.

The curvature distribution for river meanders has been found normal for natural rivers (Thakur and Scheidegger 1968) and applied as such in meander modelling (Langbein and Leopold 1966, Surkan and Kan 1969). Tests for normality at the 10% level were unsuccessful in rejecting the normal distribution for all reaches except Hole-J. The pre-recognition cf Hole-J as a joint controlled passage makes this finding of possible significance. The presence of a normal curvature distribution in a lunar rill (Thakur and Scheidegger 1968), and the similarity in related statistical properties between erosional coastlines, a crenulated divide and river meanders (Ghosh and Scheidegger 1971) suggest that normality as such is of less significance than the departure from normality.

Chang and Toebes (1970) considered the higher statistical moments of the curvature series. Their distributions were found to be generally leptokurtic (peaked), with an increase in kurtosis with discharge to a maximum defined by geology; higher for bedrock and Wisconsin till than for Illinoian till. There was no regularity in skewness. The present distributions are all leptokurtic (Table 4) and, except for Hole-J, all had kurtosis less than four (a normal distribution has kurtosis of three). The leptokurtic nature of the distributions agrees with Chang and Toebes. Hole-L, Hole-E and Hole-J do show an increase in kurtosis downstream, although discharge does not increase markedly. Shaft and Mains have a major, discrete inlet between them, and do not exhibit this property. The finding of increasing kurtosis with discharge by Chang and Toebes is probably largely a result of increasing meander scale (decreasing curvature), while the sampling interval remained constant. This would limit curvature to a smaller range, thereby increasing the general peakedness of the distribution. The bedrock meanders with the high kurtosis might be a reflection of larger meanders. The high kurtosis of Hole-J, however, is considered to represent fracture control of the passage form. Where a passage follows a fracture, there will be only relatively small changes in direction. This relatively invariant population will be supplemented by major changes in direction where either an orthogonal fracture set assumes control, or the stream reverts to the maximum gravitational gradient, which might be discordant with the structural axis.

Caution must be exercised in interpretation of kurtosis as Baker (1968) pointed out. The moment is not necessarily a simple measure of peakedness in either an absolute sense or relative to a normal distribution

The skewness (Table 4) could not be readily interpreted, and a running calculation of skewness from Hole-L gave results varying from -4.4 to 5.1. Skewness is probably a very form-sensitive parameter, but remains uninterpretable in the present situation.

The distribution of change in curvature has not been studied previously. The kurtoses of these distributions were calculated (Table 5), and some passages (eg. Mains, Hole-J) have high values, a result of wide tails and narrow peaks. This means a relatively constant change in curvature (perhaps constant curvature with measurement error), with occasional larger changes (maybe structual control). The passage plans indicate little apparent similarity between these two reaches, except the possibility of two discrete scales of meandering. The very broad distribution of change in curvature recorded for Polld may be a result of inherent irregularity of form (Fig 19), or use of too great a sampling interval.

(iii) Sinuosity

The sinuosity of the reaches is variable (Table 6), ranging from virtually straight (eg. Culla) to somewhat tortuous channels (eg. Hole-E). There is no relation to gradient (Table 2). An association was observed, however, with passage height. This implies sinuosity is somewhat time dependent, and that the meanders gradually increase in definition as the passage downcuts. Hole-L has increased its bend size and sinuosity compared to its predecessor Hole-H (or Hole-N) despite the major cutoff in the upper reaches of Hole-L. The caves with very low sinuosity (Culla, Caths, Polld) might be considered immature meanders, whose form is still actively evolving.

Individual bends, defined as half wavelengths, allowed calculation of the average sinuosity from all the bends in a given reach. The channel length is the same for each method of calculation, and it is the line defining "valley length" which varies. In the overall sinuosity this is a straight line, while the individual bends constrain this line to the meander axis. Thus, the degree of disparity between the two values is an indication of the extent to which the series departs from a simple, well-aligned series of meanders. Culla, Caths and Polld show little difference between the two measures (Fig 20a), perhaps because they are relatively young and are not yet independent from their linear, initiating structures. Hole-E is a major departure and this can be observed in the planform (Fig. 12). This difference appears superficially to represent a superimposed, large waveform, but the fracture distribution in this reach make structural control more plausible. The protochannel above this section was found to be complex in form because of these intersecting fractures.

The dispersion of direction and curvature series are clearly of some form significance (Speight 1965a, Langbein and Leopold 1966, Ghosh and Scheidegger 1971, Ferguson 1975) and variance has been related to stream sinuosity (Thakur and Scheidegger 1970, Ferguson 1977). Variance was calculated as a linear statistical moment from the direction series standardised to zero mean. The mean vector strength (R) is the circular second moment and it was compared to the variance, its linear counterpart (Fig 20b). A relationship clearly exists, although not all points conform to this. Ferguson (1977) showed that sinuosity could be calculated from the variance of normally distributed direction series. The present distributions are more or less normal (Table 3), and the sinuosity values calculated from the direction variances accorded fairly well with those actually recorded (Fig 21). In his calculations Ferguson (1977) actually proved sinuosity to be inversely related to the vector strength, although he did not recognise this association, which is far more precise than his own approximation (Fig. 21, 22). The amount of error in Ferguson's method appears to be related to the departure of the distribution from normality. (Table 3). This implies that the variance of the direction series contains some error which the vector strength does not. The magnitude of this effect appears to be related to the inability of linear variance to differentiate positive and negative values, and also reflects differences in the distribution of the data.

The minimum variance principle (Langbein and Leopold 1966) showed that variance of *curvature* was a minimum for a sine-generated curve. Ferguson (1977) claimed that the *direction* variance of a sine-generated curve was lower than for natural rivers of the same sinuosity. This can only be a result of information loss, for example by discretisation, because sinuosity is a direct function of the dispersion of direction.

The calculation of vector strength, R, is also possible for curvature, and change in curvature data, which are supplementary to directional information. The reciprocal of R will then yield a second and third order sinuosity; S' and S" repectively (Table 7). By taking the sum of sinuosities and the percentage of each component of the whole, a triaxial plot may be drawn which may classify meander patterns (Fig 23a). The general trend of the plot shows higher sinuosity to be associated with lower curvature and change in curvature sinuosity. This is a closure problem caused by constraining the total to 100%, and it reduces the value of the diagram, because direction sinuosity is the dominant component. The dotted line shows the points to have an approximately equal ratio of curvature sinuosity to change in curvature sinuosity. Shaft and Hole-J have an exactly similar ratio, although their absolute sinuosities are different. Caths and especially Polld lie well off this line.

Figure 23b clarifies this relationship by plotting S' against S" and contouring the associated direction sinuosity. Polld, Shaft and Culla fall well away from the cluster of points, and this could represent some unspecified external control. The diagram also suggests that there is a specific curvature and change in curvature sinuosity associated with maximum direction sinuosity.

Ghosh and Scheidegger (1971) recognised the ambiguity of sinuosity and proposed the "degree of wiggliness" as a supplementary statistic based on curvature. Criticism levelled at the inadequacy of either sinuosity (eg. Hey 1976) or wiggliness (Ferguson 1977) in isolation is inappropriate. Channels of low sinuosity (<1.6) can only have low wiggliness, whereas higher sinuosity can be accompanied by high or low wiggliness (Ghosh and Scheidegger 1971). Figure 24 shows the relation of wiggliness to sinuosity and is a further attempt to classify planform. A rough division of the plot produces three groups: Hole-E; Mains and Shaft; and Hole-L and Hole-H. The curvature is relatively sensitive to measurement error (Ferguson 1977), and therefore the approximate nature of the figure must be recognised.

(iv) Structural Control

Study of the relation of the passage to fractures was noted in the preceding chapters to be a difficult problem. An inconclusive visual comparison was made in the absence of any viable statistical test (Table 3). It was noted above that irregularities in direction distributions were not always associated with structural elements. The initiating fractures are overtly important only when they have complex pattern (eg. Hole-E), or the passage is relatively immature. The development of cutoffs through fractures is an important structural effect, but the only clear example of this in a developed state (Fig. 14, Hole-N and Hole-L) shows the present stream passage to be oscillating once more. Fractures, therefore, appear to only direct trend in passages, unless their control is particularly strong (Hole-J, for example).

The division of the bearing, curvature and change in curvature series from Hole-L and Hole-E into elements with joints present and those without (therefore theoretically free) proved a somewhat arbitrary procedure (Table 7). This was largely due to the imprecision of joint locations, and the necessity of assigning neighbouring elements to one joint, if it occurred between them. The two sample KS test did not suceed in differentiating the two samples. Although the test is able to compare samples of different sizes, where the two samples varied widely the test was clearly biased, and therefore unsuitable.

The skewness and kurtosis of the samples were inspected, however (Table 8). Skewness was always consistent in sign for the two samples, although the values differed somewhat. In curvature (dB) and change in curvature (ddB) the skewness was always lower for the free samples. Kurtosis varied widely, but was fairly consistent between the two samples. These data were considered unimportant at the low level of resolution.

All reaches, except Culla, were accepted as freely meandering under Ongley's (1968) criterion, that the distribution of frequency of occurence of each number of recorded bearings in each degree interval (0-180°) approximated a Poisson distribution (Table 9). Although Culla is relatively inauspicious in its meandering, it is not considered to be manifestly structurally controlled. The low sinuosity of the reach rendered it unsuitable for this test. The method was modified to limit the range across which the test was made to that of the sample, in which case, no reaches were designated structurally influenced.

An assumption of the test is that any fracture would precisely constrain the passage. This is known not to be true, and therefore, the distribution was tested with five degree intervals (Table 11). The results are very different. There are many distributions of bearings on the circle which may approximate a Poisson distribution yet still exhibit structural control, and vice versa. There is no justification of the Poisson distribution in terms of the freely meandering form.

The possible relation between width and passage direction was not examined because pilot plots showed irreconcilable scatter. Also, the bearings are not a quantity compatible with width in the regression model.

(v) Meander Properties

(a) Series

The relation of radius of curvature to width was found to vary from a minimum of two to a maximum of infinity. No regularity was observed in the relationship. This may reflect either insensitivity of discretisation to this property, or its absence from the cave meanders. Previous work has never calculated this for a series, chosing well-formed meanders individually.

The increasing dispersion of curvature as direction parted from the mean (Surkan and Kan 1969) was found not to occur, rather the opposite. Correction for the larger number of samples close to the mean direction by taking the coefficient of variation did not achieve any improvement. Ferguson (1976) claimed that this property was absent from the rivers he studied, but nevertheless, claimed

> ...that curvature changes in proportion to the local deviation of the channel from the downslope direction..., and in such a way as to turn the channel back towards that direction...(p 340)

which appears to amount to the same thing.

(b) Individual Bends

The average of the individual bend properties, together with standard deviation and coefficient of variation (standard deviation over mean) are given in Table 12. The parameters themselves are clearly not independent; for example sinuosity is fully determined by arclength and bend spacing. Hole-N probably has the greatest inherent error, and is the most consistently variable. Mains is consistently less variable than average except for orientation. Standard deviation of width and curvature show highest variablility.

The bend spacing is fairly consistent; Shaft and Mains show a downstream increase, the adjoining Gardner's reaches do not, although there is relatively little increase in discharge in the latter case. Hole-E shows great variation in orientation in response to its overall change in trend.

Polld, increasingly reccgnised as somewhat aberrant, shows high, consistent width for each bend, associated with the tightest, most irregular meanders. Width decreases substantially from Shaft to Mains despite an increase in discharge, although bendspacing and arclength do increase. There is an apparent increase in meander size along Shaft (Fig 16), and the upper half of the reach has average bendspacing of 4.3m, while the lower half has an average of 6.0m, a value very similar to Mains. A fossil passage (Gunman's Cave) enters Shaft in the area of transition, and it is possible that the present meanders are inherited from an earlier cycle when Gunman's Cave carried more water, and the lower section of Shaft experienced a similar discharge to Mains. The width along Shaft is fairly constant and this would suggest that width has adjusted to the present erosional regime. This, and the lower width of Mains, show width to be independent of discharge. The great width and small meanders of Polld support this observation. Although width measurements were not made in Gardners, Hole-J was somewhat narrower, but no substantive statements can be made.

Table 13 gives the average bend properties of right and left bends. There are no regional consistencies, although orientation is always greater for right hand bends, a result of the bend definition procedure. In some cases it is a few rather large bends which create the overall differences, suggesting it is local rather than regional effects which are important. The dominant curvature of left hand bends for Hole-E is a result of the overall leftward trend in this reach. Polld and Hole-J show little difference between right and left bend curvature, yet have large differences in standard deviation of curvature. This could mean that the (shorter) left bends have a sharper apex, which might occur where the orientation of the hydraulic gradient is oblique to the structural gradient.

The major consistency is in width, although this was only measured from the Irish caves. Right bends have lower mean widths and higher standard deviations. This might occur if the characteristic bends consisted of wide parallel walled left hand bends and a narrowing down to a minimum width in the right hand bends. This seems more plausible than a narrow, irregular, right hand bend.

The erosion of cave walls is probably related to boundary layer turbulence. If the opposition of the coriolis effect to left hand flows generated increased turbulence, then greater erosion may take place. However, an enhanced flow, such as would occur in the right hand bend would seem intuitively to increase erosion. Alternatively, the Irish caves all flow in a similar mean direction (within a 50° range). If the bedrock structure were in some way anisotropic perpendicular to the mean direction (ie. along the strike), this may cause preferential erosion on the lefthand turns.

Minor inlets were infrequent and discrete, the limestone has low primary porosity and is massively bedded, therefore a preferential capture of a water table by one bend type, leading to mixture corrosion (Bogli 1971) is extremely unlikely.

The autocorrelation coefficients for the individual bend data are shown in Tables 14a and b. Hole-L and Mains show the strongest first lag correlation through all parameters. This implies some form of short-term memory existing for two bends or one whole wavelength. The orientation values commonly show a progressive decrease in correlation for several lags (eg. Mains, Shaft, Hole-J). This memory effect may be related to large scale periodicities, but more likely reflects the localisation of structural controls.

Difference between right and left bends is confirmed if first lag anticorrelation occurs (eg. Culla: orient, Hole-N: meandb). However, if right-left contrast has been observed and first lag correlation occurs, then a false averaging of right-left properties has been made (eg. Hole-E:meandb).

A downstream trend in width is suggested by the autocorrelation values, which show a strong memory (eg. Mains, Culla) and intimations of periodicity (Fig 25). Only Caths shows the strongly periodic autocorrelation which suggests the right-left contrast is valid for the whole series. A regression of order against raw width data showed Mains and Culla to have highly significant downstream decrease in width (r^2=.19 and .48 respectively). Polld and Caths showed a significant downstream increase in width (r^2= .064 and .11 respectively). The downstream decrease in Culla was caused largely by the U-shaped passage cross section, and a marked fall in stage during the survey period. However, subsequent remeasurement showed a genuine downstream decrease in width. A test autocorrelation on the width data showed a strong suggestion of periodicity, supporting the association between width and bends.

The autocorrelation of sinuosity for Mains indicates a downstream trend , while Caths shows rapid extinction. Hole-N shows strong alternations in mean curvature, which might have stemmed from the closure onto Hole-L, which on average necessitated increasing left hand curvatures, except that the left hand bends have lower average curvature. Intuitively, standard deviation of curvature is relatively independent of arclength and sinuosity, and their joint periodicity in Hole-N shows some consistency of form, peridic about four bends or two wavelengths. Hole-J shows rapid loss of memory after two lags. This may represent the minor oscillation of the passage along straight reaches, interspersed with occasional major perterbations. The more typical autocorrelation, however, is some slight short-term memory, with subsequent insignificant oscillations about zero.

Similar form in autocorrelation between variables suggests that they are correlated, and this was tested by constructing correlation matrices (Table 15). The individual bend parameters are not independent of one another, such as the relation of mean curvature and standard deviation curvature, although mean and standard deviation width are independent. Arclength and bendspacing are correlated as is expected. Sinuosity is strongly dependent only on arclength, because arclength increases faster than bendspacing at higher sinuosities.

Mean bend curvature is related to the arclength and bendspacing in the Irish caves, and to bendspacing and sinuosity in the Gardners' reaches. This suggests curvature variation with meander size. For larger meanders, the mean curvature is lower, and the greater the difference between arclength and wavelength (greater sinuosity), the greater the curvature. This accounts for the standard deviation of curvature being related to sinuosity, but less to the arclength and not at all to bendspacing. The relation of sinuosity to orientation in the Irish data is intriguing, because orientation is not a quantity whose effect is necessarily contingent on its magnitude. Orientation was found to be important only because a few extreme values were creating the significant correlations. However, there remains the possibility of anisotropic erosional regime in the caves.

There was only a weak relation of width to bendspacing and arclength, and in no individual reaches were these significant. The mean width was correlated with mean and standard deviation curvature, a relation which proved important only for Shaft, Culla and Polld on closer inspection. This result supports the possibility of the ratio of radius of curvature to width being a constant. A calculation of radius of curvature over mean width for the individual bends (Table 16) showed Polld and Shaft to be closest to the alluvial ratio of 2-3. These reaches have been singled out previously as demonstrative anomalies in certain aspects of their width and meander wavelength. Mains shows great variablility in the ratio for its right hand bends.

The standard deviations of width and curvature were correlated for the same reaches, which probably indicates common irregularity of these data. The inverse correlation of width with bendspacing stemmed from Culla, and the relation to sinuosity from Mains, Culla and Polld. The relation between width and bend orientation in Mains, Shaft and Caths may represent bedrock control. The limited extent of these relations suggests that local controls are important.

The correlations found amongst individual bend properties were generally predictable. However, the curious width relations, and some of the form implications were somewhat unexpected.

The frequency distributions of bend characteristics showed no regularity (Table 17-21). The distribution of width was the most apparently 'normal'. This means that variability is not uniformly distributed about some ideal bend, and therefore considerations of average series properties should be treated with caution.

(vi) Meander Evolution

The development of Hole-N to Hole-L (Fig 14) has been associated with a marked increase in overall sinuosity and channel length, despite the major cutoff in the upper reaches. This channel loss has reduced the overall increase in sinuosity (Table 6) to 6%, compared to 10% for the individual bend sinuosity. This proved to be a response to increasing bend arclength (7% increase), because bend spacing decreased by only 2% over the same period. Bend spacing is constrained, however, because an increase would necessitate either extinction of a full wavelength and elongation along the channel, or else a marked increase in the complexity of bend orientation. The latter has not occurred (Table 12), but there has been the loss of half a meander cycle (37 compared to 38 bends). This provides a justification for the use of linear wavelength, rather than arclength in the classification of meanders.

The mean curvature of bends has increased, but is affected by changes in sampling interval attendant on the closure of Hole-H onto Hole-L. The average orientation of individual bends has veered slightly left, compassing the mean direction. This suggest a possible leftwards migration.

Figure 14 allows a qualitative migration model to be developed. It is characterised by downstream propogation of bends, and, as suggested by mean bend properties (arclength and mean curvature), the increase in sinuosity is caused partially by increasing internal complexity, and an increase in meander belt width.

The autocorrelation statistics show the sequence of individual bends to lose periodicity during evolution (Table 14b, eg. sinuosity, arclength and orientation). There has been an increase in short term memory (one lag), however, suggesting that changes are not uniformly distributed over the reach, but localised in a non-periodic manner. The memory of orientation is markedly more periodic for Hole-N, and this may reflect a gradual weaning from structural constraints.

Unfortunately, the crudity of bend definition and the complexity of propogation precluded definition of evolutionary axes (Hickin 1974).

(vii) Stationarity

The 100 point, running sample from Hole-L and Hole-E showed some fluctuations about the mean and vector strength (Figure 26). Hole-E produced a marked trend in mean direction once it became included in the running sample. This justifies its severance from the Hole-L series. A decreasing vector strength over the first few lags shows an increase in downstream sinuosity, perhaps caused by the major cutoff in the early stages (Figure 14).

Linear trend in the bend characteristics was tested for by regression. The trends were all insignificant (except for Culla:mean width). The linear regressions on complete data series, testing for trend only, identified significant trends in the widths as noted above. Thus the bearings may be deemed 'weakly self-stationary'. More sophisticated tests of stationarity are described below.

(viii) Markov Properties

Tables 22, 23 and 24 summarise the transition matrices and its derivatives. There is a gradual loss of the order of the Markov process through the transformations. This is related to meander arclength, which, given constant sampling interval, affects the extent of runs of sign. Although Shaft has low average bendspacing, it is the larger meanders noted previously which may be producing a strong memory. This shows the danger of the averaging of series properties in such a form as the transition matrix. The test for stationarity did not indicate this change in form, however.

The strength of the Markov property is significant for all Deviation From Mean series, but Polld is not significantly Markovian in curvature, again probably a result of its short bend spacing. Only Mains and Hole-L retain the Markov property into change in curvature. These two series also have the lowest coefficients of variation for mean curvature of individual bends. Arclength is not the most significant factor at this level of transformation.

The test for stationarity was accepted at a higher significance level than the other tests, because the series were divided into three parts which were not truncated to full cycles. In shorter series, this could create a significant, but spurious rejection. Table 22 shows Mains, Caths and Polld as non-stationary, but only the former is probably of sufficient length to render the test effective. Hole-L and Hole-H are both nonstationary in curvature. Perhaps the slightly higher Chi^2 value for Hole-L is a reflection of the cutoff in its upper segment. Mains and Hole-J are non-stationary in change in curvature. Again, Hole-J is a fairly short series and possibly misleading. The non-stationarity of Mains is manifest only in deviation from mean and change in curvature series, perhaps implying some alternating dependence in transformation.

The symmetry test did not succeed in illustrating subtle asymmetry. This is partly because of the numerical structure of the test. Caths, Polld and Mains respectively appeared to be markedly more asymmetrical in the three transformations.

The limiting vector (one of the identical rows of the limiting matrix) is never in opposition to the symmetry vector in its imbalance, yet neither is it a direct transform. The limiting vector shows the proportion of the series in the particular states, in contrast to the symmetry vector which is concerned with conservative transitions. Hole-L shows a symmetrical vector with an asymmetrical limiting vector in Table 22. This implies very short fluctuations from the positive to negative state. The same table shows Hole-E and Hole-J with different limiting vectors and identical symmetry vectors.

Table 25 compares the transition and transition proportion matrices across the transformations. The increase is expressed as number gained and proportion gained (marked x). The transition matrix allows reference to the number of transitions gained (or lost), while the proportion matrix provides a comparative reference. The correlation diagonal and anticorrelation diagonal of the proportion matrix show equal gain and loss, because between them they make up the unit value of the complete matrix. The transition from deviation from mean direction to curvature would involve no change in the matrix of a perfectly aligned series of meanders. The departure from this ideal form is a result of bends (runs of sign of curvature) occurring without compassing the mean direction. The alteration from curvature to change in curvature will involve a doubling in the anticorrelation diagonal in a wave with maximum curvature at apices and minimum at inflection points (such as the sine or sine-generated curve). The anticorrelation diagonal, therefore, indicates the number of cycles over the series, while the correlation diagonal the relative length (conservatism) of bends. The latter can be readily split into right and left bend components.

The results show the Irish data to be uniformly better aligned than the Gardners reaches, because there is a smaller increase in the anticorrelation diagonal. The misalignment is manifest in the form of Hole-E, and especially Hole-J (Figs. 12 and 13), but in Hole-L and Hole-H it is suggested that minor fluctuations in curvature superimposed on large bends have caused this misalignment. The correlation diagonal shows the average relative length of bends defined in the two transformations. Mains and Caths have actually increased average bend length in individual cells, a result of compassing mean direction without an associated change in curvature. Hole-J shows a major loss in length because it has long runs away from the mean of bends with low amplitude and curvature. Polld shows a great loss also, perhaps an effect of its very small bends, although the loss of the Markov property in curvature eliminates this reach.

The relative size of positive (++) and negative (--) elements in the correlation diagonal is of little form significance. It shows the bends which have lost most length, which may be a reflection on arclength or curvature relative to sampling interval.

The second change in transitions can only be applied to Mains and Hole-L which retain the Markov property in change in curvature. These reaches are the closest to the expected change in anticorrelation values of two and also exhibit the greatest shortening, presumably because enough memory (run length) persisted after transformation.

The Markov properties provide some useful insight into form behaviour, but classification into positive and negative states entails a substantial loss of quantitative information. However, no grounds are available for deciding how changes might further be classified, for example into great and small.

(ix) Spectral Analysis

Spectra were calculated at 15% ratio of maximum lag over length of series. The resulting peaks on the spectra are summarised in terms of the bend wavelength (2x bendspacing) and arclength frequency distributions (Tables 19 and 20). It is clear that there is little useful similarity between the two sets of results.

More suitable use of spectra was made in testing for stationarity in Hole-L by splitting it into two halves. An F-test was made on pairs of spectral estimates from the two series. The degrees of freedom for such a test are given by record length over maximum lag, multiplied by a constant contingent on the filter employed in the analysis. The program BMD02T employs a Hamming filter, the constant for which is 2.516 (R. Baldwin pers. comm.).

There is less periodicity in Hole-L(1) the upstream segment (Fig 27), which may result from the cutoff. Four estimates were found to be significantly different at .05. Hole-H and Hole-L were alsc compared (Fig 28), but no significant differences were established. Hole-H showed greater discrete periodicity (peakiness) in the spectrum, again possibly a reflection of the cutoff.

The higher Nyquist frequency (3m wavelength, the minimum resolved) for Hole-H reflects the greater error in surveying. The difference between Hole-L(1) and (2) may reflect the lower average curvature for this upper reach. This would produce a more significant reading error as a proportion of the series. The spectrum is certainly more heavily biased towards the higher frequencies for the upper reach.

(x) Scallops

The results of calculations based on scallop measurements are presented in Table 26. The calculated velocities are not unreasonable values. Culla did appear somewhat low, possibly indicating lithological control of scallop length. Highly "anomalous" scallops (or flutes) of over 80cm length were noted in Culla, especially at a transition in rock type. Although Curl (1974) rejected chemical control of scallop length (ie. the Schmidt number), he accepted that rock texture could well account for some variation in size, as was suggested by Goodchild and Ford (1971). Allen (1971) emphasised the importance of "defect" markings, which develop from irregularities in substrate texture. The "passive" markings, he considered far more unusual, but succeeded in forming them in a flume when sufficient flow length and time was allowed.

The velocity values do increase above the stream, suggesting that higher flows are actively developing these scallops. Culla shows relatively little increase in velocity, but the marked increase in width above the present channel implies contemporary erosion at this level. Any change in hydrology responsible for this would almost certainly have been recorded as a contrast in scallop length. Flotsam was noted in the roof of Culla, which suggests that higher flows may cause a marked backing-up of water, and this would lower velocity at high flows. Caths shows an increase in velocity with height above the stream, but exhibits parallel walls.

The tentative calculations of flow depth, through reversal of the Manning equation were all substantially lower than the level at which the scallops were recorded. This could be readily interpreted as confirmation of fossil scallops, but the flow depths are completely unrealistic. The discharge figures conform with low flow data determined from incidental flow data from Smith et al (in Tratman 1969), except for Shaft where quite high flows are implicated. The calculated values fall in a low flow range.

The calculated depth figures indicate that much is amiss in the procedure. Curl (1973, pers. comm.) demonstrated misgivings concerning the Manning equation by showing that over equilibrium scallops (steady flow), n was highly flow dependent. The relationship of roughness elements to Reynolds number results in very high roughness at lower flows, falling off rapidly as Reynolds number increases. Where non-steady flow occurs, there is no indication of what an acceptable value for n might be. Calculations from Chow (1959) gave an n of .03 to .06. A calculation of flow depth independent of the Manning equation, showed very similar results (Parker, 1977 pers. comm.), indicating that the value of .025 selected was reasonable.

Other possible sources of error are with slope and width, assuming the flow velocities calculated from the scallops to be reasonable. Slope is always below the regional dip down which the caves flow, which is consistent with lower downstream rates of erosion. The width may be an incorrect parameter, if the scallop-forming depth is low and the bed creates substantial flow resistance.

CHAPTER V

DISCUSSION

The immediate implications of the results have been covered in the preceding chapter, and consideration is given here to problems stemming from these results. It has been difficult to place the results in the context of earlier work, partly because the methodology was developed specifically to avoid past errors. There is also very little information available on cave meanders.

Wavelength-width relationships of this, and earlier work are summarised in Figure 29 and Table 1. The only result readily comparable with the present data in this relationship is from High (1970) for Culla. This is approximately half the present finding, although the survey methods were similar. He explicitly chose a reach in which "...the published survey...showed no meanders present..." and only covered three meanders. The present data show six bends with wavelength of six or less metres and the widths are fairly similar. The present results lie midway between High, and Deike and White's (1969) determination from the published survey. Some individual bends showed wavelengths at Deike and White's higher value, which implies that the wavelengths on the survey are accurate, but only above some minimum resolution, while High lost the larger wavelengths, because of the short reach surveyed. It is quite probable that other studies drawing on published surveys have also filtered meander bends in a similar way, often to deliberately avoid "anomalous" bends.

The problem of using survys is well illustrated by Deike and White with a width of 2.7m for Culla (cf. 70cm), although one reach of Polld which they surveyed does appear to conform to the present finding, where Polld lies close to their line.

The remaining four reaches, lie well away from any previous work and show an inverse relationship of wavelength to width, significant at .05. Yalin's (1972) hypothesis clearly canot be dominant in these streams. However, no explanations are known, which can account for these results. Width also showed no relation to discharge, downstream decrease, and properties apparently contingent on the sense of a bends. These findings must be explained in terms of palaeohydrology, and contemaporary, erosional regime.

The significance of palaeohydrology was illustrated by Shaft, where meander bendspacing changes, apparently in response to Gunman's Cave, now abandoned and hanging 3.5 metres above the present stream. The lower part of Shaft appears to have maintained the form of these ancient meanders, but width has adjusted to high values throughout. Water now entering at the First Waterfall used to flow the length of Shaft (Fig 30a), while Gunman's original water has been diverted elsewhere. The Shaft meanders have not migrated significantly compared to Mains.

These sections of cave have witnessed major changes in flow regime, but present-day hydrochemistry mat also have influenced passage form (Fig 30b). Shaft has water chemistry characteristic of slow percolation at baseflow, and very little erosion can be taking place compared to Mains where aggressive water is encountered. In high flow the dilution of Shaft suggests a swallet source, when highly aggressive water flows along the passage. This agrees with the scallop evidence, that high flow velocities (and possibly discharges) are those effective in eroding this section of cave.

The width of the present caves represents an equilibrium between lateral and vertical erosion, because there is no effective lateral depositional mechanism, except ultimately passage collapse. Solutional erosion depends upon the ratio of solvent viscosity and the diffusivity of the solute in the particular solvent; giving, as a ratio, the Schmidt number. The concentration gradient is also important. In cave streams, the rapidity with which unsaturated water can be introduced to the limestone, and saturated water removed, effects erosional behaviour, and is contingent on turbulence, which is a function of stream velocity and roughness (Allen 1971). Solution is a finite process and ceases at saturation, but corrasional processes are not so limited, provided sediment can be moved.

In most cave streams, sediment occupies parts of the floor. In Gardners Gut, the material was largely sand sized, but shale cobbles were more characteristic in Ireland. Sand, as the more mobile agent, probably accounts for a large part of the erosional processes in Gardners Gut, although solutional weakening of in situ limestone particles may well be important. The prominence of the weak, residual beds, does imply solutional weakening. However, the cobbles are far less mobile, although the bedrock beneath them was found to support somewhat subdued scallops, which suggests that the material is either transitory, or else preventing erosion of the floorr. The cobbles probably protect the floor from solutional processes by decreasing flow velocity and the rate of solvent exchange. Thus, in Shaft for example, solution will be widening the channel, and the underfit nature of the stream probably means the clastic material is relatively immobile. Mains does have occasional reaches of cobbles, but the flow regime will probably move these fairly regularly. This would allow the floor to erode faster. The downstream decrease in width is probably a reflection of the exhaustion of solutional potential. This may also account for the downstream decrease in passage height.

The existence of chemical, rather than flow thresholds in cave streams, and the failure of Shaft meanders to migrate, suggest that incision is critical to meandering in the solutional environment. In a few locations notable migration of meanders was associated with short, steep reaches. It is possible that flow properties responsible for alluvial meandering also create chemical effects which lead to differential erosion similar to that active in alluvial meanders. The change from smooth to rough turbulent flow will enhance solution rates, providing there is some erosional potential in the stream. This is because a faster rate of solvent exchange will occur at the water-rock interface. The change from subcritical to supercritical regime may also enhance the rate of intake of carbon dioxide at the water surface.

Passages showing an increase in width downstream (eg. Caths, Polld), have either an increase in corrasional erosion or the addition of rejuvenating, aggressive water. The paradoxical difficulty of explaining downstream width increase, led to a more detailed inspection. Polld did show a downstream decrease, until an inlet joined the stream, where an increase in width occured. This may be because either the influent water is aggressive, or the mischungscorrosion effect. The mechanism of mischungscorrosion has been accepted (Howard 1966), but its importance remains unclear (Curl 1966a). It has been an implicit assumtion above, that lithology itself is uniform throughout a reach and region. This need not be true and the effect of variaticn on solution may be particularly marked. Rauch and White (1970) showed the importance of lithology in controlling cavern development.

The difference between right and left bend width is difficult to rationalise in terms of solutional processes. The possibility of a directional constraint on shelving noted above, suggests some form of microstructure may exist within individual bends.

The radius of curvature over width proved greater than for surface rivers (Table 16). The closest reaches were Shaft where width was enhanced due to clastic protection, and Polld. The bed material of Polld was not recorded, but a coarse clastic cover is recalled, in which case protection may be occurring. This means that small cave streams are characteristically narrower than surface rivers in relation to bend curvature. The apparent accordance with bedrock meanders is superficial, however, because solutional erosion does not depend on discharge alone, but is a continual process. Caths showed a steady increase in width over half its length and little further change. Caths is a strongly scalloped stream, which is indicative of solutional erosion. The most likely explanation of increasing width is that downstream, increasing clastic protection of the bed has enhanced lateral erosion.

CHAPTER VI

CONCLUSIONS

In karst areas with only moderately dipping limestones and concordant hydraulic gradient, caves may develop sinuous reaches. Fractures control cave trend, but the meandering processes appear to be increasingly dominant in determining passage form as the cave develops. Eventually, fractures are of local importance only, although they may promote stream cutoffs.

A comparison of passage and joint orientation has to remain qualitative, because the samples are generally not independent, and joints are not an homogeneous population in terms of either location or axis. The distribution of passage curvature (change in direction) may be non-normal and may show high kurtosis under some conditions of structural control.

The discrete orientation series can be readily obtained from a straightforward field survey. Information contained in the series can be resolved by transformation, eg. individual bend form and Markov properties. The initial sampling interval has to be determined by both the need for an adequate description of the series and the necessity of keeping measurement error within tolerance levels. The

Nyquist frequency and test for the Markov property provide tests for both these constraints.

Sinuosity of any wiggly line is equal to the reciprocal vector strength. In like manner, the 'sinuosity' of curvature and change in curvature can also be calculated for a particular series, to produce three indeces closely describing form. Sinuosity shows an increase with age in the present caves, but eventually should reach an equilibrium level.

The width of cave passages is a reflection of erosional regime and history. Where solutional processes are dominant, width reaches a maximum set not by loss of erosional capacity, but rather by rate of downcutting (or ultimately passage collapse). If bed sediment prevents active downcutting in a stream, width may be somewhat greater. In an unbranched conduit, width decreases downstream, because of loss of solutional potential. Width is inversely related to wavelength in the present study, but this may be a result of local controls on the former. The wavelength probably has some relation to discharge, but is highly conservative over time compared to width.

The development of meandering appears to demand active incision of the cave stream. Corrosional processes are in part controlled by flow behaviour, and therefore erosional regimes maintaining evolutionary meanders may be a response to flow structures similar to those responsible for alluvial meanders.

A specific case of meander evolution showed some downstream propogation of bends, and an increase in complexity, sinuosity and belt width, although no great change in wavelength.

The Markov chain treament of meander series provides a convenient method for form description, especially for changes with differencing. The reduction of a series to binary data, however, renders the process somewhat summary.

The curvature series is considered the most suitable for spectral analysis, but the spectrum is not comparable with discrete meander descriptions. The method is useful for comparative purposes, however.

Reasonable velocity estimates can be made from scallop lengths. A scallop-forming discharge can then be computed, and although calculations show flows compatible with hydrochemistry, problems arise in interim values of depth.

Finally, these quantitative studies have demonstrated the importance of the field context, not only on location, but in the intimate manipulation of the data.

Inevitably, much work remains to be done, for the results raise more problems than they solve. Most of this novel methodology has not been applied to alluvial rivers. This would provide some measure of control for the present results. Questions concerning the erosional regime of solutional, fluvial systems remain, although techniques are already available which could provide some approximate answers.

TABLES

Source	Situation	Coef.	Exp.
Inglis (1949)	Alluvial Rivers	6.6	0.99
Leopold, Wolman (1957)	Alluvial Rivers	6.5	1.1
Leopold, Wolman (1960)	Alluvial Rivers	10.9	1.01
Carlston (1965)	Alluvial Rivers	16.	1.
Tinkler (1971)	Alluvial Rivers	12.43	
Ferguson (1975)	Alluvial Rivers	11.0	1.14[1]
Ferguson (1975)	Alluvial Rivers	20.0	1.04[2]
Dury (1976)	Alluvial Rivers	9.76	1.109
Ackers, Carlton (1970b)	Flume	6.84	0.760
Inglis (1949)	Incised Rivers	2.06	1.27
Tinkler (1971)	Incised Rivers	19.70	
Deike (1967)	Cave Passages	5.5	
Ongley (1968)	Cave Passages	5.5	
Deike, White (1969)	Cave Passages	6.8	1.05[3]
Deike, White (1969)	Cave Passages	8.2	0.92[4]
Baker (1973)	Cave Passages	7.4	1.15

Notes:

1. Calculated from the autocovariance
2. Calculated from the spectrum.
3. Missouri Caves
4. Other Caves.

Table 1.
Values of the Coefficients and Exponents Relating Meander Wavelength to Width in a Power Curve.

Variable	Reach				
	Mains	Shaft	Culla	Caths	Polld
Width					
Mean:	0.65	0.89	0.70	0.51	1.14
Stan.Deviation:	0.16	0.12	0.17	0.13	0.15
Skewness:	4.04	0.16	0.41	1.19	0.99
Kurtosis:	2.68	5.33	2.49		18.36
Gradient:	0.014	0.026	0.024	0.031	0.019

Table 2.
Width and Gradient Data for the Irish Caves.

Bearing	Reach																
(°)	Mains		Shaft		Culla		Caths		Polld		HoleL		HoleH	HoleE		HoleJ	
	B	J	B	J	B	J	B	J	B	J	B	J	B	B	J	B	J
0- 9:											2		1	3	6	1	6
10- 19:											2			3	3	3	7
20- 29:											1		2	5	3	3	8
30- 39:											6		7	2	4		6
40- 49:											9		6	4	5	1	8
50- 59:											14		6	8	2		
60- 69:											4		7	4			
70- 79:	2										7		12	1			
80- 89:	3		4								7	1	8	12		1	
90- 99:	10	2	5								11	9	12	7		4	
100-109:	5	6	2	1							10	17	7	11		1	
110-119:	4	12	11	10							10	32	9	4		8	
120-129:	6	14	7	6							14	11	8	10	1	6	
130-139:	7		6	3							19	1	12	4	1	5	
140-149:	6		9	2					1		13		13	1		7	4
150-159:	4	1	7	2			1		2		8		16	6	7	9	11
160-169:	6	1	9		4		3		2		12		23	3	6	5	10
170-179:	8	5	8	2	2		8		5		14		9	2	22	9	14
180-189:	7	24	7	4	13		7		4		18		11	6	6	13	6
190-199:	7	16	8	35	10	3	5	6	8	2	9		21	8	3	6	7
200-209:	8		8	13	23		19	8	6	6	15		9	4	3	6	8
210-219:	9		5	1	29	3	15	10	11	16	12		5	1	4	1	6
220 229:	9		12	1	25	4	4	9	9	3	13		6	1	5	2	8
230-239:	10		5	1	8		10		7	2	4		4	1	2	3	
240-249:	13		2		4		13		6		7		2			2	
250-259:	4		3		4		8		4		4						
260-269:	7		2		1		5		7		3	1	3				
270-279:	7	2	3		2				8		2	9	5				
280-289:	3	6	5	1			2			1	1	17	1				
290-299:	1	12	3	10			1		4	4	2	32	5				
300-309:	2	14	1	6					2		1	11	1	5			
310-319:	1								1			1		1	1		
320-329:											1						4
330-339:											1			1	7		11
340-349:														3	6	1	10
350-359:											2			7	22	1	14
Mean Bearing:	186		169		203		209		219		143		148	88		149	
Test for Normality																	
Kolmogorov Stat. :	.058		.049		.064		.064		.044		.031		.044	.135		.095	
D at .10:	.100		.106		.109		.121		.131		.077		.080	.108		.123	

Reject Ho only for HoleE

Table 3.
Frequency Distributions of Passage Direction(B) and Jointing(J) with Mean Direction.

Change in Bearing (°)		Reach								
		Main	Shaf	Cull	Cath	Poll	HleL	HleH	HleE	HleJ
-100	-91:									
-90	-81:		2			1		1		0
-80	-71:	2	3			2	1	0		0
-70	-61:	1	3		2	1	4	3	1	1
-60	-51:	2	3		0	2	5	2	3	2
-50	-41:	5	2	1	3	2	10	6	5	2
-40	-31:	7	5	1	5	7	14	14	8	3
-30	-21:	16	7	8	7	8	15	22	10	5
-20	-11:	24	20	23	13	7	31	23	12	6
-10	-1:	25	21	23	21	7	38	54	21	24
0	9:	19	25	32	24	12	59	38	34	26
10	19:	15	13	17	9	17	27	24	12	15
20	29:	12	10	7	7	7	12	20	8	5
30	39:	7	7	2	5	1	18	7	5	3
40	49:	6	4	1	2	2	10	3	3	2
50	59:	3	2		1	6	10	9	3	1
60	69:	3	2		1	1	5	3	0	0
70	79:	1	1			0	1	0	2	1
80	89:		0			1	1	1		
90	99:		1			0				
100	109:					0				
110	119:					1				

Test for Normality

Kolmogorov Statistic:	.052	.077	.034	.070	.076	.054	.060	.086	.133
Rej. Ho at 0.10?	No	No	No	No	No	No	No	No	Yes
Skewness :	.69	-2.6	2.4	1.3	-0.8	-2.1	-0.6	-1.0	2.4
Kurtosis :	3.29	3.89	3.51	3.84	3.64	3.17	3.87	3.71	5.58

Table 4.
Frequency Distributions of Curvature, Test for Normality and Skewness and Kurtosis.

Change in Change in Bearing (°)	Reach								
	Main	Shaf	Cull	Cath	Poll	HleL	HleH	HleE	HleJ
-140 -131:					2				
-130 -121:					0				
-120 -111:					0				
-110 -101:					0				
-100 -91:	1	1			1				1
-90 -81:	0	1			1		1		0
-80 -71:	1	1			1	1	3	1	0
-70 -61:	1	0		3	4	4	2	2	1
-60 -51:	2	4		2	0	4	3	2	2
-50 -41:	5	6	1	3	8	10	6	3	5
-40 -31:	6	8	13	7	5	25	21	10	9
-30 -21:	13	8	13	7	5	25	21	10	9
-20 -11:	20	17	20	9	8	32	25	10	6
-10 -1:	34	22	30	17	5	39	42	25	22
0 9:	26	17	25	15	6	48	41	21	21
10 19:	10	16	13	14	8	30	25	18	8
20 29:	14	8	11	9	8	23	19	15	6
30 39:	4	2	6	3	4	15	18	3	6
40 49:	3	7		2	4	6	7	3	1
50 59:	5	5		3	2	4	2	2	1
60 69:	2	2		4	2	2	0	1	4
70 79:	1	1			2		1	1	1
80 89:		2			2		1		
90 99:		1			3				
100 109:					0				
110 119:					1				
Kurtosis :	4.11	3.58	2.73	2.95	3.14	3.28	3.83	3.50	3.99

Table 5.
Frequency Distribution of Change in Curvature and Kurtosis.

Reach	Variable					
	Vector Strength	Direction Variance	Calculated Sinuosity	Overall Sinuosity	Mean Bend Sinuosity	Wiggliness
Mains	0.546	63	1.75,1.83	1.83	1.37	4102
Shaft	0.605	55	1.62,1.65	1.65	1.16	4071
Culla	0.936	8	1.07,1.07	1.07	1.04	
Caths	0.874	15	1.14,1.14	1.14	1.11	
Polld	0.800	25	1.25,1.25	1.25	1.15	
HoleL	0.490	78	2.00,2.04	2.04	1.44	6507
HoleH	0.527	71	1.88,1.90	1.90		5875
HoleE	0.427	89	2.24,2.34	2.34	1.24	3319
HoleJ	0.637	59	1.67,1.57	1.57	1.20	
HoleN				1.92	1.32	

Note: Calculated sinuosity is arranged as:
Sinuosity from Ferguson (1977), Reciprocal Vector Strength

Table 6.
Measures of Sinuosity.

Reach	Direction	Curvature	Change in Curvature
Mains	1.831	1.123	1.111
Shaft	1.653	1.154	1.185
Culla	1.069	1.033	1.046
Caths	1.144	1.078	1.140
Polld	1.251	1.210	1.438
HoleL	2.040	1.107	1.100
Holeh	1.899	1.103	1.112
Holee	2.343	1.108	1.108
Holej	1.569	1.089	1.125

Table 7.
Sinuosity Calculated as Reciprocal Vector Strength of Direction, Curvature, and Change in Curvature.

Bearing (Degrees)	B				Reach and Transformation								
						dB				ddB			
	HoleE		HoleL		(Degrees)	HoleE		HoleL		HoleE		HoleL	
	J	F	J	F		J	F	J	F	J	F	J	F
0- 9:			1			1							
10- 19:	1	8	5	11	/-70 -61:	1		1	3	1			3
20- 29:		3	2	10	/-60 -51:	2	1	5		1	1	1	3
30- 39:	2	3	3	19	/-50 -41:	2	3	4	6	1	1	1	9
40- 49:	3	6	8	10	/-40 -31:	6	2	8	6	1	2	1	5
50- 59:	4	0	6	5	/-30 -21:	9	2	5	11	5	4	2	19
60- 69:		1	4	7	/-20 -11:	6	7	18	16	2	8	6	20
70- 79:	6	6	2	8	/-10 -1:	14	9	17	21	3	7	12	33
80- 89:	4	3	2	11	/ 0 9:	20	17	23	37	9	15	6	36
90- 99:	4	7	2	9	/ 10 19:	12	4	13	15	5	15	12	23
100-109:	1	3	5	7	/ 20 29:	5	3	5	7	6	11	7	18
110-119:	7	8	4	11	/ 30 39:	4	2	8	11	8	7	5	12
120-129:	1	4	3	16	/ 40 49:	3		3	8		3	3	4
130-139:		1		14	/ 50 59:	3		5	1	1	2	2	4
140-149:	1	6	1	8	/ 60 69:				1	1	1		2
150-159:	2	4	3	9	/ 70 79:	3		1			1		
160-169:	4	5	3	13	/ 80 89:					1			
170-179:	2	7	3	15									

Table 8.
Comparison of Frequency Distributions of Bearings (B), Curvature (dB), and Change in Curvature (ddB) for Jointed (J) and Free (F) Segments of Passage.

Series		HoleL		HoleE	
		Skewness	Kurtosis	Skewness	Kurtosis
B	J:	1.93	3.20	-2.17	2.22
	F:	0.56	2.63	-0.46	1.72
dB	J:	-5.50	2.40	-4.48	3.29
	F:	-6.14	4.39	-8.87	3.74
ddB	J:	-1.58	2.04	-4.36	2.31
	F:	-5.28	2.07	-5.96	2.02

Table 9.
Skewness and Kurtosis of Frequency Distributions in Table 8.

Reach		Number of Bearings per 1° Class					KS Statistic	Reject Ho At .10?
		0	1	2	3	>3		
Mains	Obs:	87	50	34	5	4	0.044	No
	Exp:	79	65	27	7	2		
Shaft	Obs:	85	66	23	4	2	0.008	No
	Exp:	86	63	23	6	1		
Culla	Obs:	117	27	22	7	7	0.15	Yes
	Exp:	90	62	22	5	1		
Caths	Obs:	118	36	18	3	5	0.083	No
	Exp:	103	58	16	3	0		
Polld	Obs:	116	44	17	3	0	0.028	No
	Exp:	111	54	13	2	0		
HoleL	Obs:	41	63	47	22	7	0.025	No
	Exp:	44	62	43	20	7		
HoleH	Obs:	65	53	31	20	13	0.071	No
	Exp:	50	64	41	18	6		
HoleE	Obs:	89	57	31	3	0	0.033	No
	Exp:	88	63	22	5	1		
HoleJ	Obs:	107	55	13	3	2	0.013	No
	Exp:	104	57	15	3	0		

Table 10.
Test for Poisson Distribution of Frequency of Occurence of Number of Bearings per Degree Class.

Reach		No.of Bearings in Five Degree Class											KS Test	Reject Ho at .10?
		0	1	2	3	4	5	6	7	8	9	10		
Mains	Ob:	0	3	7	7	4	4	6	3	1	0	1		
	Ex:	1	2	5	7	7	6	4	2	1	1	0	0.083	No
Shaft	Ob:	0	5	5	8	5	9	2	1	0	1	0		
	Ex:	1	3	6	8	7	5	3	2	1	0	0	0.056	No
Culla	Ob:	14	5	4	2	1	0	3	1	0	1	5		
	Ex:	1	4	7	8	7	5	3	1	1	0	0	0.039	No
Caths	Ob:	10	8	2	3	3	5	1	0	0	3	1		
	Ex:	2	6	9	8	6	3	1	1	0	0	0	0.28	Yes
Polld	Ob:	7	10	4	5	3	3	2	1	1	0	0		
	Ex:	3	8	9	8	5	2	1	0	0	0	0	0.17	Yes
HoleL	Ob:	0	0	0	2	4	5	4	6	8	0	7		
	Ex:	0	0	1	2	3	5	5	5	5	4	3	0.10	Yes
HoleH	Ob:	0	0	0	4	7	5	5	4	5	3	3		
	Ex:	0	0	1	3	4	5	6	5	4	3	2	0.063	No
HoleE	Ob:	3	5	6	1	10	5	1	3	2	0	0		
	Ex:	1	4	7	8	7	5	3	1	1	0	0	0.12	Yes
HoleJ	Ob:	7	6	7	5	3	2	2	3	1	0	0		
	Ex:	2	6	9	8	5	3	1	1	0	0	0	0.14	Yes

Table 11.
Test for Poisson Distribution of Frequency of Occurence of Number of Bearings per Five Degree Class

Variable					Reach				
	Mains	Shaft	Culla	Caths	Polld	HoleL	HoleN	HoleE	HoleJ
Meandb	21.90	24.04	11.10	16.52	26.21	19.07	18.81	19.12	15.47
(Deg)	9.94	12.41	5.91	8.73	13.02	8.00	9.98	9.86	8.54
	.45	.51	.53	.53	.50	.42	.53	.52	.55
Sdevdb	14.81	18.53	8.46	19.14	24.03	17.05	16.39	14.55	15.93
(Deg)	7.91	11.18	4.74	8.75	14.13	6.78	8.34	8.74	9.82
	.53	.60	.56	.46	.59	.40	.51	.60	.62
Bendsp	6.13	5.36	6.70	7.52	4.74	6.70	6.83	5.48	6.52
(m)	2.32	2.41	3.61	3.55	3.07	2.79	3.51	2.22	9.82
	.38	.45	.54	.47	.65	.42	.51	.40	.43
Sinuos	1.39	1.25	1.04	1.10	1.15	1.44	1.31	1.22	1.19
	.38	.31	.06	.07	.16	.43	.34	.26	.22
	.27	.25	.06	.06	.14	.30	.26	.21	.18
Arclen	8.64	6.89	6.98	8.25	5.50	9.71	9.07	6.89	7:94
(m)	4.07	3.97	3.70	3.79	3.74	4.62	5.09	3.59	4.17
	.47	.58	.53	.46	.68	.48	.56	.52	.52
Orient	184.8	170.6	202.2	207.9	220.0	142.0	144.2	100.3	142.8
(Deg)	43.16	41.96	14.16	13.01	26.01	47.08	52.88	80.30	42.50
	.23	.25	.07	.06	.12	.33	.37	.80	.30
Meanwi	62.20	88.02	78.05	52.66	115.2				
(cm)	13.50	9.92	13.08	7.50	9.70				
	.22	.11	.18	.14	.08				
Sdevwi	8.09	10.71	6.21	9.42	6.59				
(cm)	4.84	8.52	4.41	6.25	4.35				
	.60	.80	.71	.66	.66				

Sdev is standard deviation, dB is change in bearing, Bendsp is the linear bend spacing, Arclen is bend arclength, Orient is the bend orientation, and wi is channel width.

After each parameter the values are arranged as:

Series Mean
Series Standard Deviation
Series Coefficient of Variation.

Table 12.
Mean, Standard Deviations and Coefficient of Variation of Individual Bend Properties.

Variable	Reach								
	Mains	Shaft	Culla	Caths	Polld	HoleL	HoleN	HoleE	HoleJ
Meandb	23.23	24.61	10.97	14.44	26.09	19.35	19.86	16.27	15.59
(m)	-20.67	-23.42	-11.23	-18.59	-26.32	-18.80	-17.71	-22.19	-15.37
Sdevdb	15.06	16.69	9.10	18.80	21.63	17.42	17.58	13.40	13.35
(Deg)	14.58	20.52	7.82	19.47	26.22	16.69	15.13	15.78	18.22
Bendsp	5.61	5.95	6.45	8.57	5.12	7.11	6.90	5.66	6.81
(m)	6.61	4.73	6.95	6.48	4.40	6.29	6.76	5.28	5.26
Sinuos	1.37	1.16	1.04	1.11	1.15	1.44	1.32	1.24	1.20
	1.40	1.35	1.05	1.10	1.14	1.44	1.29	1.21	1.18
Arclen	8.00	7.07	6.69	9.38	6.00	10.18	9.27	7.29	8.63
(m)	9.23	6.69	7.27	7.13	5.05	9.24	8.86	6.46	7.33
Orient	196.3	182.8	206.5	212.2	230.1	150.1	153.6	111.6	145.1
	174.2	157.5	198.0	203.6	210.9	133.9	134.3	88.1	140.8
Meanwi	60.69	87.11	77.13	49.47	114.5				
(cm)	63.59	88.99	78.98	55.84	114.8				
Sdevwi	8.88	10.91	8.00	11.82	7.10				
(cm)	7.37	10.50	4.43	7.01	6.12				

After each keyword the values are arranged as:
Right hand bend average
Left hand bend average KEY: see Table 12.

Table 13.
The Properties of Individual Bends, Right and Left.

Reach	Meandb	Sdevdb	Bendsp	Sinuos	Arclen	Orient	Meanwi	Sdevwi	Lag
Mains	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0
	-.10	.25	-.35	.51/	.08	.38	.51/	.26	1
	-.16	-.19	-.02	.28	.16	.08	.41	.12	2
	.05	-.07	-.02	.37	.29	.21	.16	-.01	3
	-.21	-.21	-.07	.15	.00	.52/	-.08	.00	4
Shaft	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0
	.13	.02	-.00	.08	.23	.33	.52/	.01	1
	.30	-.09	.03	-.25	-.20	.34	.09	-.14	2
	-.08	-.38	.08	-.14	.14	-.09	-.28	-.18	3
	-.03	.08	.55/	.34	.55/	-.26	-.33	-.20	4
	.15	-.09	.14	.00	.35	-.63/	-.23	.06	5
Culla	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0
	.02	-.08	-.08	.16	-.08	.11	.75/	-.05	1
	-.16	-.08	-.39	-.09	-.44/	.04	.50/	.06	2
	.13	.10	.27	-.13	.26	.15	.34	-.35	3
	-.38	.02	.24	-.07	.28	.17	.26	-.05	4
	.01	-.04	.04	-.05	.06	-.44/	.27	-.14	5
Caths	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0
	-.20	.25	-.10	.20	-.05	.10	-.24	.13	1
	-.23	-.14	-.14	-.48	-.06	.11	.34	.39	2
	.01	-.29	-.10	-.26	-.14	.19	-.19	.02	3
Polld	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0
	-.03	.11	-.19	-.13	-.26	.09	.38	-.24	1
	-.14	-.07	.03	.09	.01	-.12	.18	.12	2
	.22	.04	-.05	-.02	-.04	-.45/	.10	-.15	3
	-.09	-.09	-.16	-.33	-.22	.08	-.09	-.16	4

/ signifies autocorrelation significant at .05
KEY: see Table 12.

Table 14a.
Autocorrelation of Bend Properties: Continued overleaf.

Reach	Variable						
	Meandb	Sdevdb	Bendsp	Sinuos	Arclen	Orient	Lag
HoleL	1.00	1.00	1.00	1.00	1.00	1.00	0
	.19	.22	.07	.35/	.28	.33/	1
	-.05	-.22	-.13	.07	.12	-.10	2
	-.07	-.04	-.02	.32/	.20	-.18	3
	.07	.11	.10	.25	-.02	-.18	4
	-.22	.12	-.18	.07	-.10	-.07	5
	-.28	-.11	-.25	.14	.04	.04	6
	-.13	-.11	-.03	.26	-.17	-.16	7
HoleN	1.00	1.00	1.00	1.00	1.00	1.00	0
	-.17	.02	-.05	.22	.10	.35/	1
	.24	-.22	.06	-.47/	-.22	-.07	2
	-.05	-.13	.10	-.43/	-.17	-.06	3
	.11	.03	.26	.05	.29	-.12	4
	-.06	.30	-.10	.49/	.17	-.44/	5
	.08	.02	-.09	.04	-.02	-.28	6
	-.21	-.45/	.06	-.42/	-.13	-.07	7
HoleE	1.00	1.00	1.00	1.00	1.00	1.00	0
	.13	-.00	.28	-.08	.11	.27	1
	.09	-.17	-.07	-.30	-.17	-.23	2
	.05	.00	.24	.09	.19	-.08	3
	.13	.09	-.05	-.02	-.05	-.13	4
	.04	.02	-.42/	-.02	-.39/	-.11	5
HoleJ	1.00	1.00	1.00	1.00	1.00	1.00	0
	-.03	-.02	.09	.19	-.02	.63/	1
	-.12	-.48/	.04	-.30	.07	.12	2
	-.00	.05	-.24	-.00	.01	.04	3

/ signifies autocorrelation significant at .05
KEY: see Table 12.

Table 14b.
Autocorrelation Coefficients of Individual Bend Properties at Twenty Percent Lag.

Irish Data

Sdevdb	.57/	1.00					
Bendsp	-.54/	-.16	1.00				
Sinuos	.24	.38/	.05	1.00			
Arclen	-.39/	-.01	.90/	.48/	1.00		
Orient	.22	.11	.00	-.25/	-.11	1.00	
Meanwi	.31/	.28/	-.25/	.06	-.20/	.08	1.00
Sdevwi	.24/	.43/	.06	.36/	.18	-.03	-.00
	Meandb	Sdevsb	Bendsp	Sinuos	Arclen	Orient	Meanwi

Gardners Data

Sdevdb	.64/	1.00			
Bendsp	-.29/	.16	1.00		
Sinuos	.35/	.45/	.15	1.00	
Arclen	-.10	.31/	.86/	.62/	1.00
Orient	-.08	-.01	.04	-.08	-.01
	Meandb	Sdevdb	Bendsp	Sinuos	Arclen

/ represents a significant correlation at .05.

Table 15.
Correlation Matrices for Individual Bend Properties.

Reach									
Mains		**Shaft**		**Culla**		**Caths**		**Polld**	
Mean	**Sdev**	**Mean**	**Sdev**	**Mean**	**Sdev**	**Mean**	**Sdev**	**Mean**	**Sdev**
27.3	72.8	5.5	3.4	17.5	18.0	17.9	13.5	3.9	2.2
8.6	5.3	6.2	4.9	14.4	13.0	10.0	4.6	3.5	1.6
17.6	50.4	5.8	4.1	15.9	15.5	13.9	10.6	3.7	1.9

Values are in Columns arranged as:
Right hand Bend
Left hand Bend
All Bends

Table 16.
Mean and Standard Deviation of Radius of Curvature over Width for Individual Bends.

Class	Reach				
Range (cm)	Mains	Shaft	Culla	Caths	Polld
40- 50	3	0	0	8	0
60	10	0	2	7	0
70	7	2	6	0	0
80	2	2	9	1	0
90	2	10	5	0	0
100	1	12	1	0	1
110	0	1	3	0	6
120	0	0	0	0	8
130	0	0	0	0	5
140	0	0	0	0	0
150	0	0	0	0	1

Table 17.
Frequency Distribution of Widths for Individual Bends.

Class	Reach								
Range (°)	Mains	Shaft	Culla	Caths	Polld	HoleL	HoleN	HoleE	HoleJ
0.0- 2.5	1	0	2	0	0	0	1	0	1
5.0	0	1	2	1	0	3	1	4	1
7.5	0	3	4	0	0	1	4	0	1
10.0	1	3	5	2	1	1	4	1	0
12.5	1	0	4	5	2	2	2	1	3
15.0	2	1	3	1	1	5	0	2	3
17.5	5	3	2	0	2	2	4	2	0
20.0	2	4	2	2	0	9	4	5	3
22.5	4	1	1	1	0	7	6	4	3
25.0	0	1	0	2	5	0	2	2	0
27.5	2	2	1	0	0	1	2	3	0
30.0	2	1	0	1	0	1	0	0	1
32.5	1	1	0	0	3	4	3	1	0
35.0	1	2	0	0	1	2	2	0	1
37.5	0	2	0	1	1	0	0	1	0
40.0	1	0	0	0	1	0	2	0	0
42.5	2	0	0	0	0	0	0	0	0
45.0	0	0	0	0	0	0	0	0	0
47.5	0	1	0	0	0	0	0	1	0
50.0	0	1	0	0	0	0	0	0	0
>50.0	0	0	0	0	1	0	0	0	0

Table 18.
Frequency Distribution of Curvature
for Individual Bends.

Class Range (m)	Reach Mains	Shaft	Culla	Caths	Polld	HoleL	HoleN	HoleE	HoleJ
4- 6	3**	4*	6	2*	8	4	2**	5	3
8	0	5	0I	2	1*	3*	6*	2	0
10	6	6*	5I	0	6*	7	6	5*	4I
12	5	6	4I	2I	3	3	6	9	2I
14	2I	2	1	3I	1	3	4	0	0I
16	4I	1	3	0I	0	7	0	2	4I
18	4I	1(*)	0	1I	0	0I	7I	0	0I
20	0I	0	1	2I	0	8I	0I	3	1I
22	0I	1	2	2	1	2I	1I	1	2
24	0I	0	2	1	0	0	0I	0	1
26	1I	0	0	0	0	0	1I	0	0
28	0I	1	0	0	0	0	2I	0	0
30	0	0	2	0	0	1	1I	0*	0
32	0	0	0	1	1	0	0I	0	0

* signifies a spectral peak
I signifies a broad spectral peak

Table 19.
Distribution of Wavelength (2 x Bendspacing) for Individual Bends, and Peaks of Spectrum of Curvature Series.

Class Range (m)	Reach Mains	Shaft	Culla	Caths	Polld	HoleL	HoleN	HoleE	HoleJ
2- 3	3**	4*	6	2*	8	4	2**	5	3
4	0	0	0	0	0*	0*	3*	0	0
5	2	8*	4I	2	6*	4	6	6*	2I
6	5	6	5I	2I	4	4	5	5	3I
7	0I	0	0I	0I	0	0	2	0	0I
8	3I	3	4	3I	0	4	0	3	1I
9	3I	1	0	1I	0	4I	3I	1	4I
10	0I	0(*)	0	0I	0	0I	2I	0	0I
11	2I	1	3	2	1	4I	1I	5	0
12	2I	1	2	3	1	3	3I	1	2
13	0I	0	0	0	0	0	1I	0	0
14	3I	0	0	0	0	4	2I	0	1
15	1I	2	2	0	0	1	0I	0	0
16	0	0	0	0	0	0	1I	0	0
17	0	0	0	1	0	4	3I	0	0
18	1	1	0	0	1	2	1I	1	1
19	0	0	0	0	0	0	1I	0*	0
20	0	0	0	0	0	0	1I	0	0

* signifies a spectral peak
I signifies a broad spectral peak

Table 20.
Distribution of Individual Bend Arclength, and Peaks of Spectrum of Curvature Series.

Class				Beach					
Range (°)	Mains	Shaft	Culla	Caths	Polld	HoleL	HoleN	HoleE	HoleJ
0- 15	0	0	0	0	0	0	0	2	0
30	0	0	0	0	0	0	0	4	0
45	0	0	0	0	0	1	2	2	1
60	0	0	0	0	0	0	0	1	1
75	0	0	0	0	0	1	1	3	0
90	0	0	0	0	0	5	4	3	0
105	1	2	0	0	0	2	5	2	0
120	0	0	0	0	0	3	2	1	1
135	2	5	0	0	0	5	2	1	2
150	3	4	0	0	0	4	1	2	3
165	4	1	0	0	0	5	5	3	4
180	1	3	2	0	1	4	6	1	3
195	3	4	6	4	3	1	3	0	1
210	3	3	12	4	3	3	3	0	1
225	3	2	5	7	4	4	1	0	0
240	3	1	0	1	6	0	1	0	0
255	2	1	1	0	2	0	0	0	0
270	0	1	0	0	1	0	1	0	0
285	0	0	0	0	0	0	0	0	0
300	0	0	0	0	1	0	0	0	0
>300	0	0	0	0	0	0	0	2	0

Table 21.
Frequency Distribution of Orientation of Individual Bends.

Reach	Transition Matrix				Transition Probability Matrix				Transition Proportion Matrix			
	++	+-	--	-+	++	+-	--	-+	++	+-	--	-+
Mains	66	10	61	9	.87	.13	.87	.13	.45	.07	.42	.06
Shaft	55	10	54	10	.85	.15	.84	.16	.43	.08	.42	.08
Culla	48	14	46	14	.77	.23	.77	.23	.39	.11	.38	.11
Caths	29	12	45	12	.71	.29	.79	.21	.30	.12	.46	.12
Polld	30	11	32	11	.73	.27	.74	.26	.36	.13	.38	.13
HoleL	109	15	111	14	.88	.12	.89	.11	.44	.06	.45	.06
HoleH	109	12	94	13	.90	.10	.88	.12	.48	.05	.41	.06
HoleE	51	8	58	8	.86	.14	.88	.12	.41	.06	.46	.06
HoleJ	41	5	45	4	.89	.11	.92	.08	.43	.05	.47	.04

Reach	Limiting Vector		Order	Chi^2 on Markov	Chi^2 on Statnry.	Symmetry Vector		Chi^2 on Symmetry
	+	-				++	--	
Mains	.49	.51	5	80.87/	5.35/	.26	.24	.08
Shaft	.50	.50	5	66.17/	1.27	.25	.25	.00
Culla	.51	.49	4	35.57/	2.80	.25	.25	.01
Caths	.42	.58	4	46.74/	3.96/	.20	.30	2.34
Polld	.49	.51	4	21.81/	9.93/	.24	.26	.05
HoleL	.48	.52	5	167.58/	.26	.25	.25	.00
HoleH	.55	.45	5	137.66/	.89	.27	.23	.27
HoleE	.47	.53	5	88.52/	8.08	.24	.26	.21
HoleJ	.43	.57	6	77.32/	5.69	.24	.26	.11

Chi^2 Test: / signifies Ho rejected at .1, .05 and .1 respectively for Markov property, stationarity and symmetry test.

1. Deviation from Mean Direction.

Table 22.
Transition, Transition Probability and Transition Proportion Matrices, Limiting Vector, Order and Test for Markov Property, Stationarity and Symmetry for Deviation from Mean Direction.

Reach	Transition Matrix				Transition Probability Matrix				Transition Proportion Matrix			
	++	+-	--	-+	++	+-	--	-+	++	+-	--	-+
Mains	49	15	65	16	.77	.23	.80	.20	.34	.10	.45	.11
Shaft	48	17	46	17	.74	.26	.73	.27	.38	.13	.36	.13
Culla	35	23	40	23	.60	.40	.63	.37	.29	.19	.33	.19
Caths	30	18	30	19	.63	.38	.61	.39	.31	.19	.31	.20
Polld	26	19	18	20	.58	.42	.47	.53	.31	.23	.22	.24
HoleL	98	34	82	34	.74	.26	.71	.29	.40	.14	.33	.14
HoleH	70	37	83	37	.65	.35	.69	.31	.31	.16	.37	.16
HoleE	48	18	39	18	.73	.27	.68	.32	.39	.15	.32	.15
HoleJ	33	17	27	17	.66	.34	.61	.39	.35	.18	.29	.18

Reach	Limiting Vector		Order	Chi^2 on Markov	Chi^2 on Statnry.	Symmetry Vector		Chi^2 on Symmetry
	+	-				++	--	
Mains	.46	.54	4	72.37/	3.45	.21	.29	.98
Shaft	.51	.49	4	27.43/	2.95	.26	.24	.02
Culla	.48	.52	3	9.75/	3.74	.23	.27	.22
Caths	.51	.49	3	6.01/	3.73	.25	.25	.00
Polld	.55	.45	2	.95	3.28	.29	.21	1.65
HoleL	.53	.47	4	39.41/	10.65/	.27	.23	.40
HoleH	.47	.53	3	38.24/	6.18/	.23	.27	.36
HoleE	.54	.46	4	15.26/	1.72	.28	.23	.54
HoleJ	.53	.47	3	4.56/	2.38	.27	.23	.50

Chi^2 Test: / signifies Ho rejected at .1, .05 and .1 respectively for Markov property, stationarity and symmetry test.

2. Curvature

Table 23.
Transition, Transition Probability and Transition Proportion Matrices, Limiting Vector, Order and Test for Markov Property, Stationarity and Symmetry for Curvature

	Transition Matrix				Transition Probability Matrix				Transition Proportion Matrix			
Reach	++	+-	--	-+	++	+-	--	-+	++	+-	--	-+
Mains	32	33	46	33	.49	.51	.58	.42	.22	.23	.32	.23
Shaft	34	28	37	28	.55	.45	.57	.43	.27	.22	.29	.22
Culla	26	32	30	32	.45	.55	.48	.52	.22	.27	.25	.27
Caths	22	26	21	27	.46	.54	.44	.56	.23	.27	.22	.28
Polld	19	22	19	22	.46	.54	.46	.54	.23	.27	.23	.27
HoleL	65	65	52	65	.50	.50	.44	.56	.26	.26	.21	.26
HoleH	60	54	57	55	.53	.47	.51	.49	.27	.24	.25	.24
HoleE	34	30	29	30	.53	.47	.49	.51	.28	.24	.24	.24
HoleJ	21	26	19	27	.45	.55	.41	.59	.23	.28	.20	.29

Reach	Limiting Vector		Order	Chi^2 on Markov	Chi^2 on Statnry.	Symmetry Vector		Chi^2 on Symmetry
	+	-				++	--	
Mains	.45	.55	2	5.87/	4.15/	.20	.30	1.61
Shaft	.49	.51	2	2.61	2.13	.24	.26	.09
Culla	.48	.52	2	.29	3.51	.23	.27	.26
Caths	.51	.49	2	1.09	.57	.25	.24	.03
Polld	.50	.50	2	.44	.55	.25	.25	.00
HoleL	.53	.47	2	3.50/	2.46	.28	.22	.62
HoleH	.51	.49	2	.23	.85	.26	.24	.03
HoleE	.52	.48	2	.23	5.02	.27	.23	.31
HoleJ	.51	.49	2	2.22	4.53/	.27	.23	.12

Chi^2 Test: / signifies Ho rejected at .1, .05 and .1 respectively for Markov property, stationarity and symmetry test.

3. Change in Curvature

Table 24.
Transition, Transition Probability and Transition Proportion Matrices, Limiting Vector, Order and Test for Markov Property, Stationarity and Symmetry for Change in Curvature

	Mains	Shaft	Culla	Caths	Polld	HoleL	HoleH	HoleE	HoleJ
++	-17	-7	-13	1	-4	-11	-39	-3	-8
--	4	-8	-6	-15	-14	-29	-11	-19	-18
++,--	-13	-15	-19	-14	-18	-40	-50	-22	-26
-+,+-	12	14	18	13	17	39	49	20	25

Transition Matrix

++	-.11	-.05	-.10	.01	-.04	-.04	-.17	-.02	-.08
--	.03	-.06	-.05	-.15	-.16	-.12	-.05	-.15	-.19
++,--	-.08	-.11	-.15	-.14	-.21	-.16	-.22	-.16	-.27
x	.90	.87	.80	.82	.72	.82	.76	.81	.71
-+,+-	.08	.11	.15	.14	.21	.16	.22	.16	.27
x	1.64	1.71	1.66	1.56	1.79	2.35	2.97	2.29	3.82

Transition Proportion Matrix

1. From Deviation from Mean Direction to Curvature

	Mains	Shaft	Culla	Caths	Polld	HoleL	HoleH	HoleE	HoleJ
++	-17	-14	-9	-8	-7	-33	-10	-14	-12
--	-19	-9	-10	-9	1	-30	-26	-10	-8
++,--	-36	-23	-19	-17	-6	-63	-36	-24	-20
-+,+-	35	22	18	16	5	62	35	24	19

Transition Matrix

++	-.12	-.11	-.07	-.08	-.08	-.13	-.04	-.11	-.13
--	-.13	-.07	-.08	-.09	.01	-.12	-.11	-.08	-.08
++,--	-.24	-.18	-.15	-.17	-.07	-.25	-.16	-.20	-.21
x	.69	.76	.75	.72	.87	.65	.77	.72	.67
-+,+-	.24	.18	.15	.17	.07	.25	.16	.20	.21
x	2.14	1.66	1.40	1.45	1.14	1.92	1.48	1.67	1.58

Transition Proportion Matrix

2. From Curvature to Change in Curvature

Values are numerical increases from matrix to matrtix, except for those marked 'x', which give relative increases.

Table 25.
Change in strength of the Correlation Diagonal (++,--) and the Anticorrelation Diagonal (+-,-+) for the Transition Matrix and the Transition Proportion Matrix in Transformation from Deviation From Mean Direction to Curvature to Change in Curvature.

	Reach						
Variable	Shaft	Culla	Culla	Caths	Caths	Caths	Gardner
Ht. above stream (cm)	1-10	1-10	40-60	1-10	40-50	90-100	1-10
Calculated Depth (cm)	2.6	0.7	0.7	2.8	4.5	5.7	2.4
Width (cm)	89	60	85	50	51	50	33
Slope	.026	.024	.024	.031	.031	.031	(?.03)
Scallop Mean (cm)	6.2	11.6	12.4	5.2	4.1	3.7	5.3
Velocity (cm/s)	54.2	22.5	23.0	60.3	80.0	90.6	52.5
Calc. Discharge (l/s)	12.4	.96	1.43	8.38	18.4	25.6	(?4.14)
Obs. Discharge	1.42^{1} ^{2} 18.9^{1} ^{3}		11.50^{3} 1.24 .71^{4} .89^{4}				

1. Discharge calculated as percentage of flow immediately downstream at first waterfall.
2. Presumed base flow.
3. Flood stage.
4. Calculated from measurements at swallet.

Table 26.
The Calculation of Scallop-forming Discharge from Scallop Length and Mannings Equation.

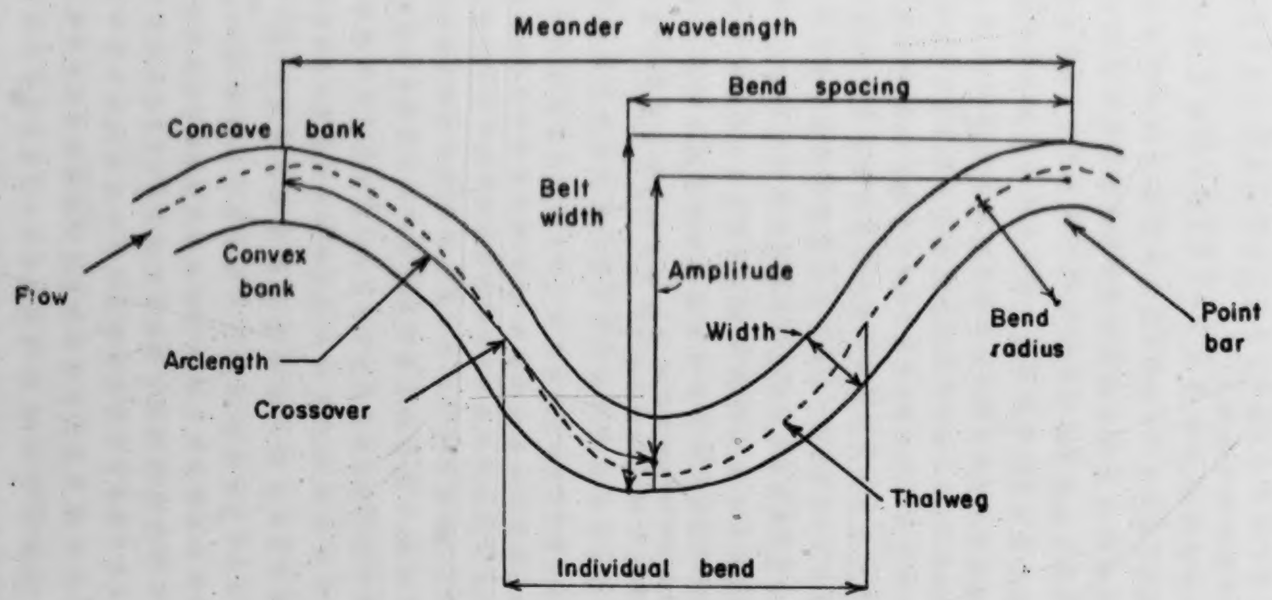
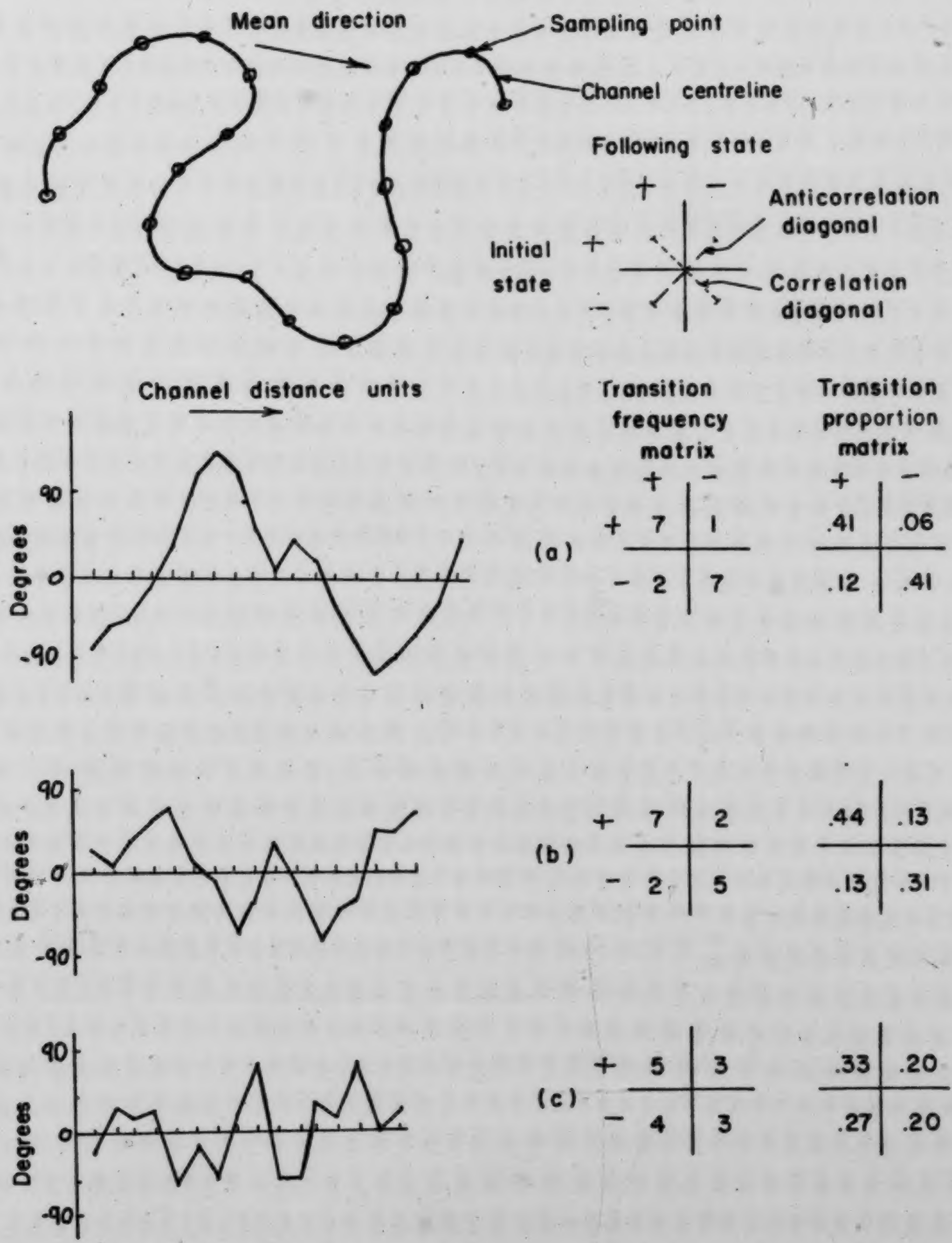

Meander wavelength
Bend spacing
Concave bank
Belt width
Convex bank
Amplitude
Flow
Arclength
Crossover
Width
Bend radius
Point bar
Thalweg
Individual bend

FIGURE I. DEFINITION DIAGRAM FOR A RIVER MEANDER


Mean direction
Sampling point
Channel centreline
Following state
+
−
Initial state
+
−
Anticorrelation diagonal
Correlation diagonal
Channel distance units
Degrees
90
0
-90
Transition frequency matrix
Transition proportion matrix
(a)
(b)
(c)

(a)

	+	−		+	−
+	7	1		.41	.06
−	2	7		.12	.41

(b)

	+	−		+	−
+	7	2		.44	.13
−	2	5		.13	.31

(c)

	+	−		+	−
+	5	3		.33	.20
−	4	3		.27	.20

FIGURE 2. DISCRETISATION OF A SHORT REACH OF CHANNEL
(a) DEVIATION FROM MEAN DIRECTION
(b) FIRST DIFFERENCES
(c) SECOND DIFFERENCES

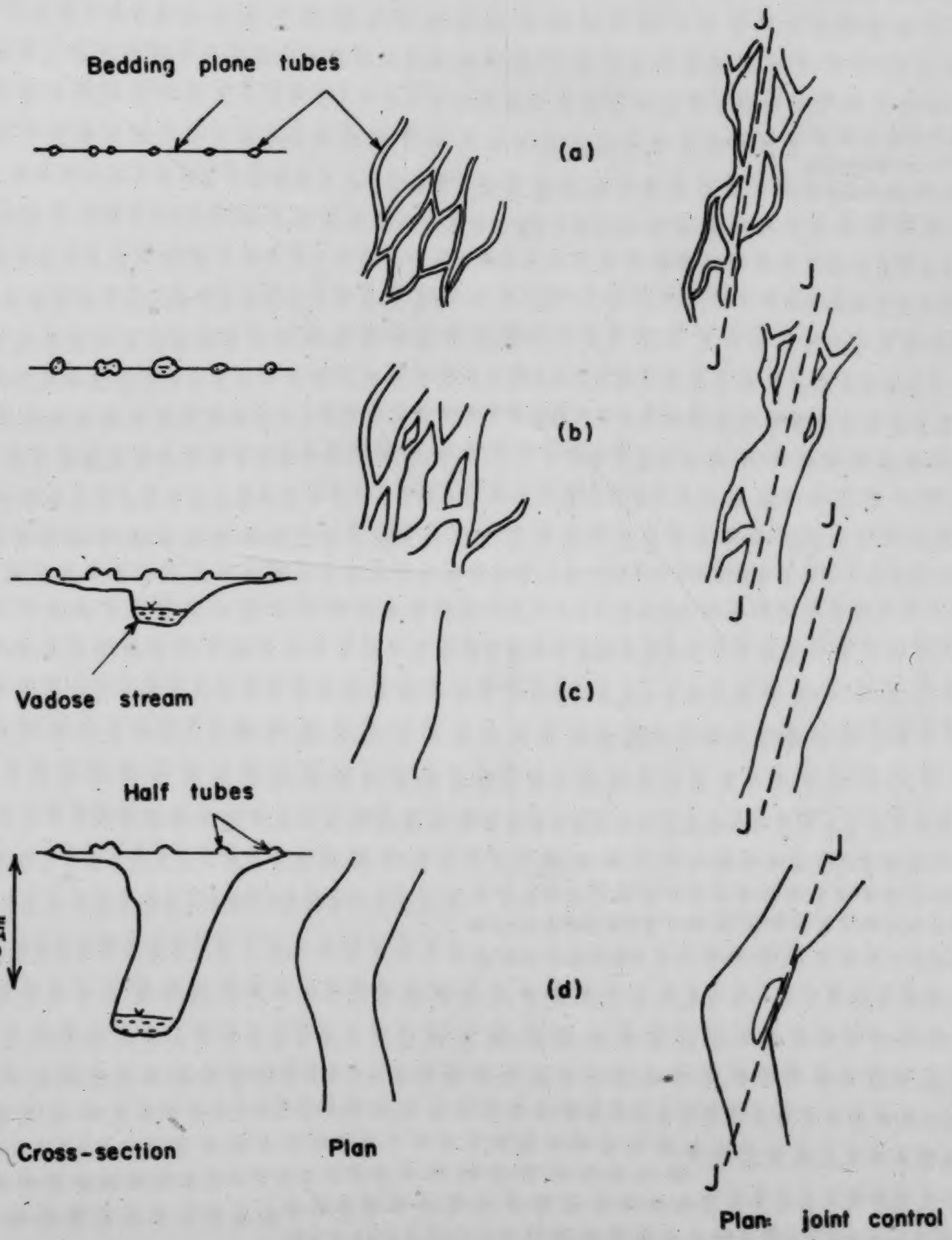

Bedding plane tubes
(a)
(b)
(c)
(d)
Vadose stream
Half tubes
~1m
J
J
J
J
J
J
J
Cross-section
Plan
Plan: joint control

FIGURE 3. EVOLUTION OF T-FORM PASSAGE TYPICAL OF COUNTY CLARE, IRELAND

(a) PERCOLATION THROUGH SMALL TUBES
(b) CAPTURE OF DOMINANT TUBE
(c) VADOSE DOWNCUTTING
(d) GROWTH OF MEANDER

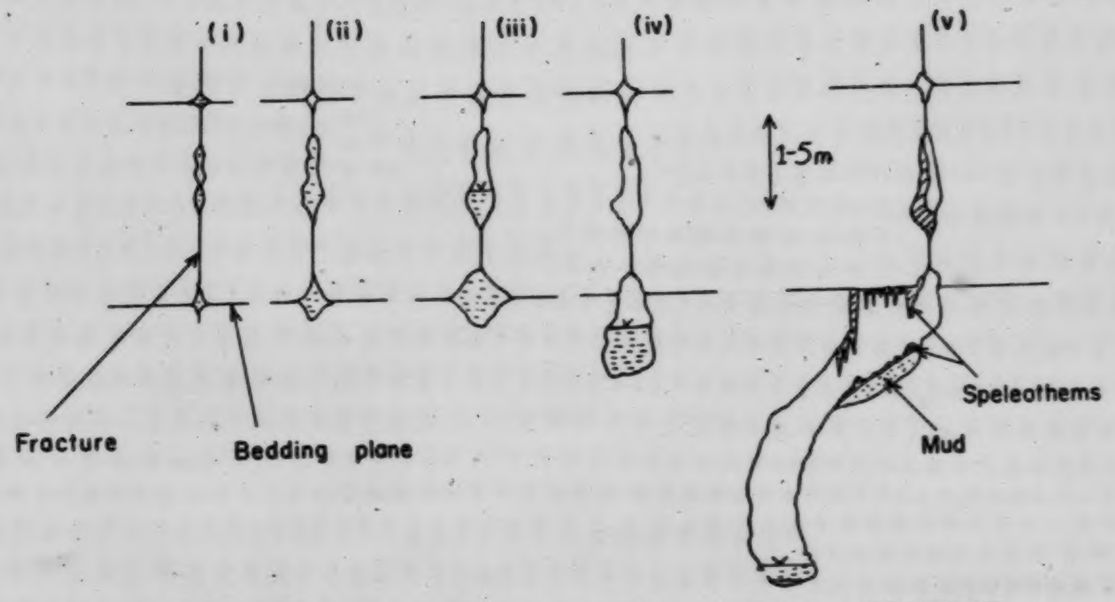

(i)
(ii)
(iii)
(iv)
(v)
1-5m
Speleothems
Mud
Fracture
Bedding plane

FIGURE 4a. EVOLUTION OF PASSAGE FROM A VERTICAL FRACTURE
(i), (ii), (iii) GROWTH ALONG LINES OF WEAKNESS
(iv) VADOSE DOWNCUTTING
(v) PRESENT SITUATION: DEPOSITION ABOVE, EROSION BELOW

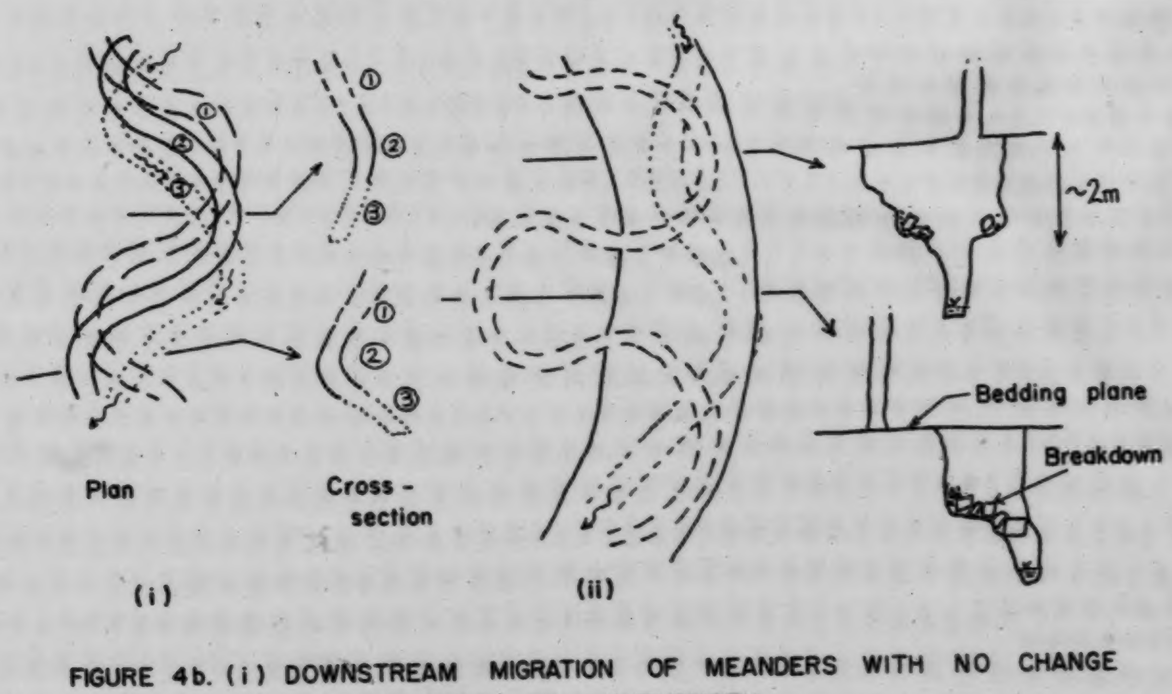

①
②
③
~2m
Bedding plane
Breakdown
Plan
Cross - section
(i)
(ii)

FIGURE 4b. (i) DOWNSTREAM MIGRATION OF MEANDERS WITH NO CHANGE IN AMPLITUDE OR WAVELENGTH
(ii) THREE MORPHOLOGICALLY DISTINCT MEANDER TYPES, BURR CAVE, WAITOMO, NEW ZEALAND

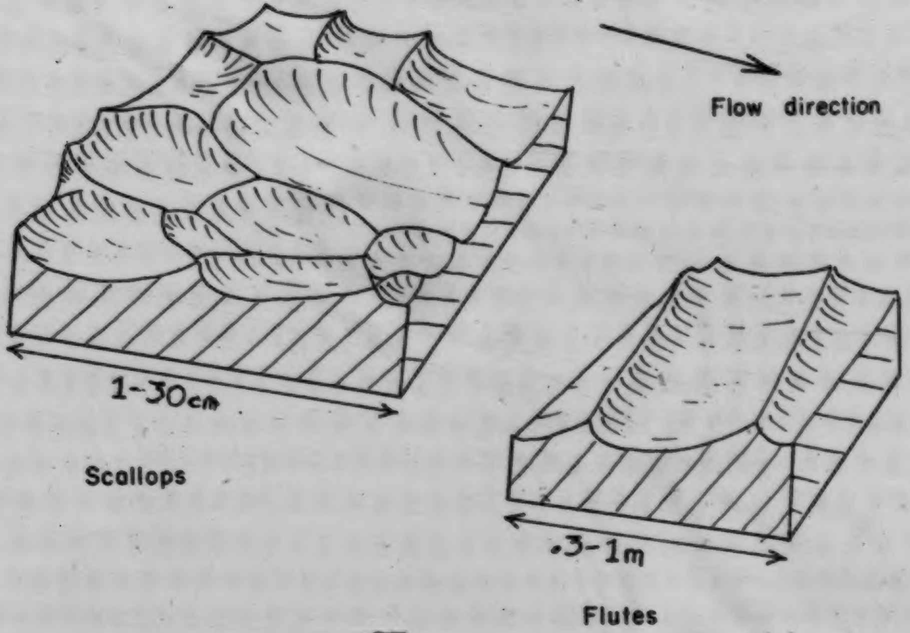

Flow direction
1-30cm
Scallops
.3-1m
Flutes

FIGURE 5a. TYPICAL ASSEMBLAGES OF SCALLOPS AND FLUTES

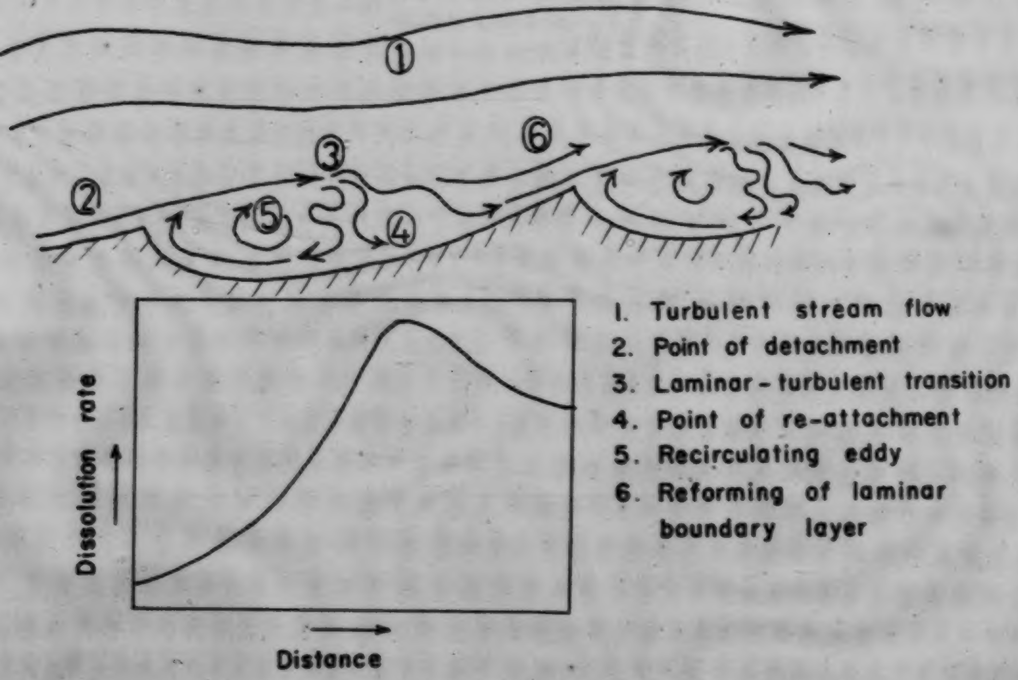

①
②
③
④
⑤
⑥
Dissolution rate
Distance
1. Turbulent stream flow
2. Point of detachment
3. Laminar - turbulent transition
4. Point of re-attachment
5. Recirculating eddy
6. Reforming of laminar boundary layer

FIGURE 5b. FLOW OVER A SCALLOP AND LONGITUDINAL DISSOLUTION RATE (after Blumberg and Curl 1974)

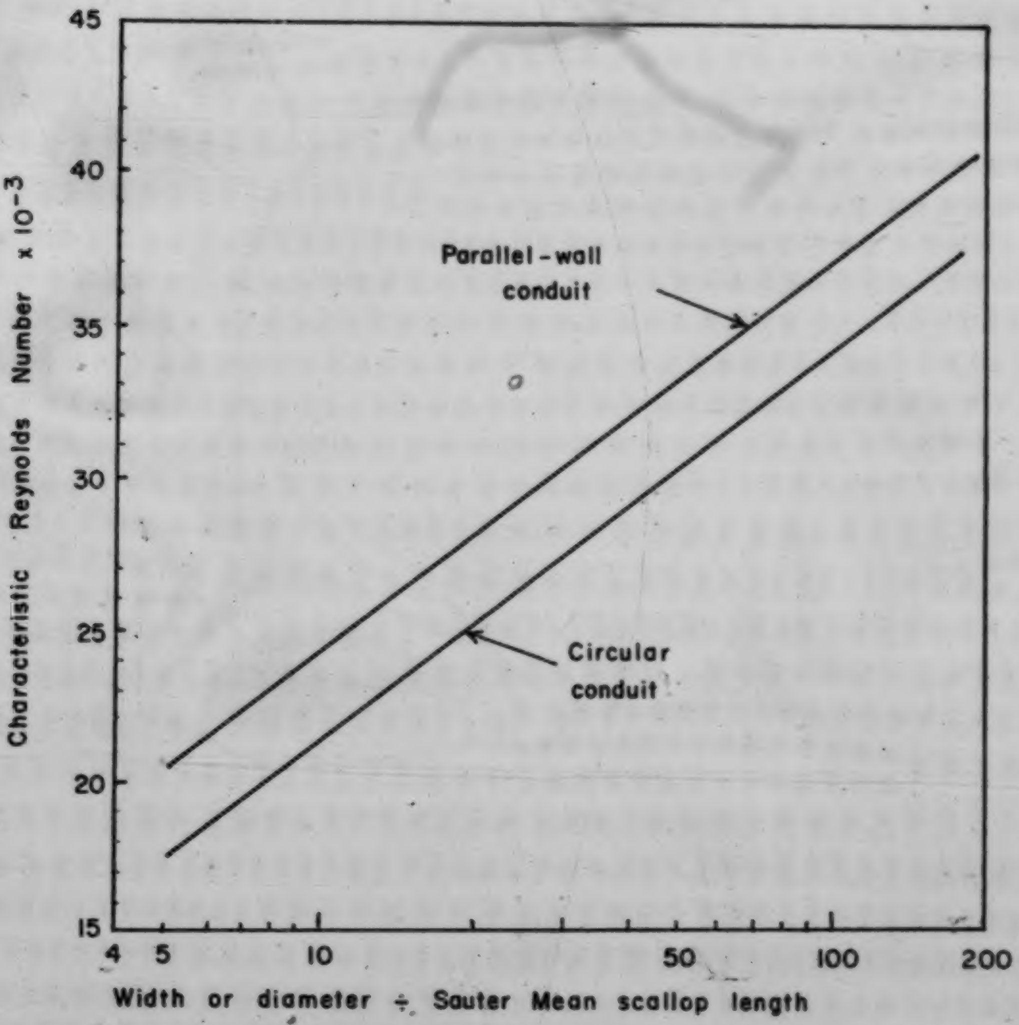
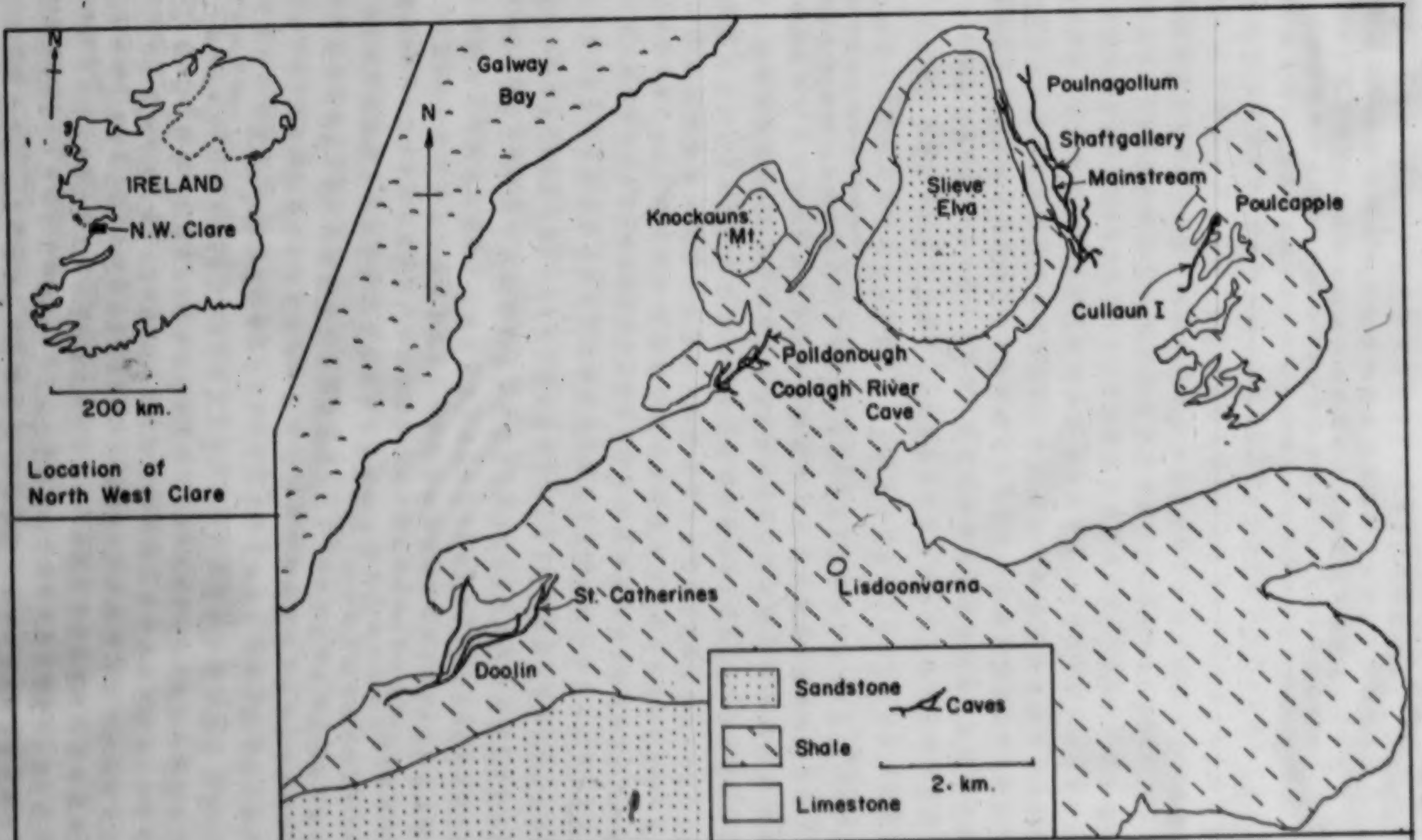


FIGURE 6. THE THEORETICAL RELATION BETWEEN CHARACTERISTIC REYNOLDS NUMBER AND THE RATIO OF CONDUIT WIDTH OR TUBE DIAMETER TO SAUTER MEAN SCALLOP LENGTH (after CURL 1974)



N
IRELAND
N.W. Clare
200 km.
Location of
North West Clare
Galway
Bay
N
Knockauns
Mt.
Slieve
Elva
Poulnagollum
Shaftgallery
Mainstream
Poulcapple
Cullaun I
Polldonough
Coolagh River
Cave
Lisdoonvarna
St. Catherines
Doolin
Sandstone
Caves
Shale
2. km.
Limestone

FIGURE 7. NORTH WEST COUNTY CLARE : GEOLOGY AND CAVES (from Caves of Northwest Clare by E.K. Tratman)

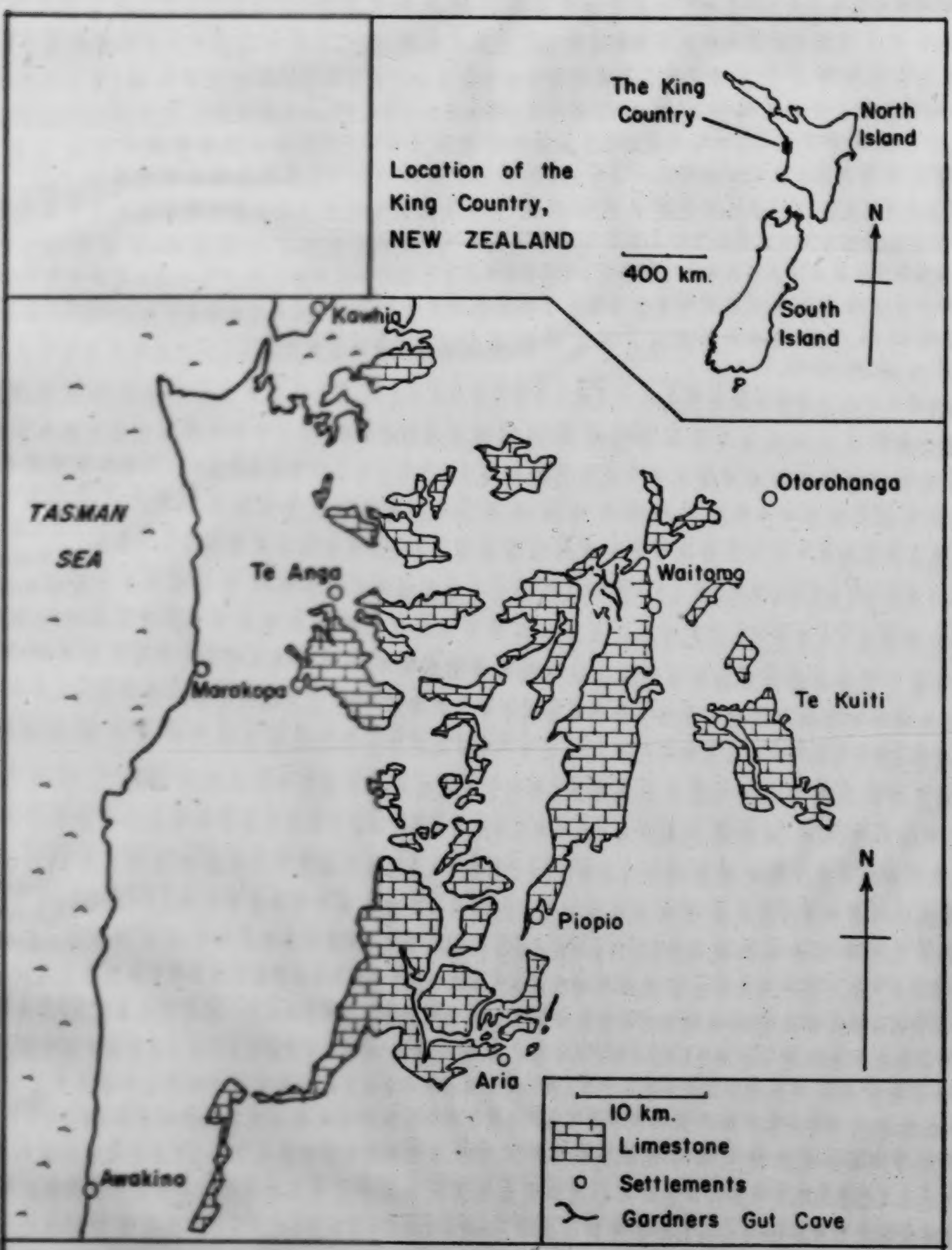

Location of the
King Country,
NEW ZEALAND
The King Country
North Island
South Island
400 km.
N
Kawhia
Otorohanga
TASMAN SEA
Te Anga
Waitomo
Marakopa
Te Kuiti
Piopio
Aria
Awakino
N
10 km.
Limestone
Settlements
Gardners Gut Cave

FIGURE 8. THE KING COUNTRY: KARST AREAS AND GARDNERS GUT CAVE
(after Kermode 1975)

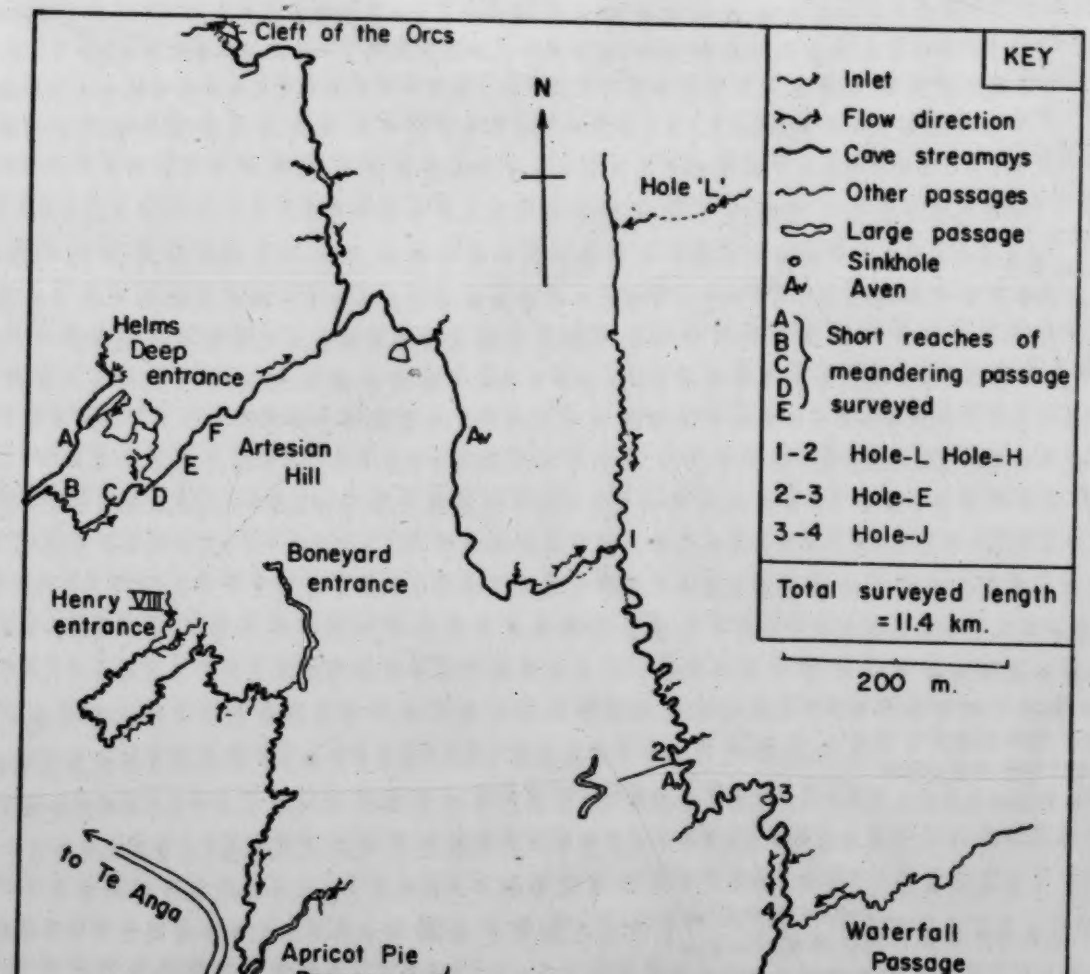

Cleft of the Orcs
N
Hole 'L'
KEY
Inlet
Flow direction
Cave streamays
Other passages
Large passage
Sinkhole
Aven
A B C D E Short reaches of meandering passage surveyed
1-2 Hole-L Hole-H
2-3 Hole-E
3-4 Hole-J
Total surveyed length = 11.4 km.
200 m.
Helms Deep entrance
A
B
C
D
E
F
Artesian Hill
Av
1
Boneyard entrance
Henry VIII entrance
2
Av
3
4
to Te Anga
Apricot Pie Passage
Waterfall Passage
Road
Exit 7
Av
Av
Av
Av
Levels
to Waitomo
Peter Lambert
~400m. to exit

FIGURE 9.

GARDNERS GUT CAVE

(from N.Z.S.S. survey)

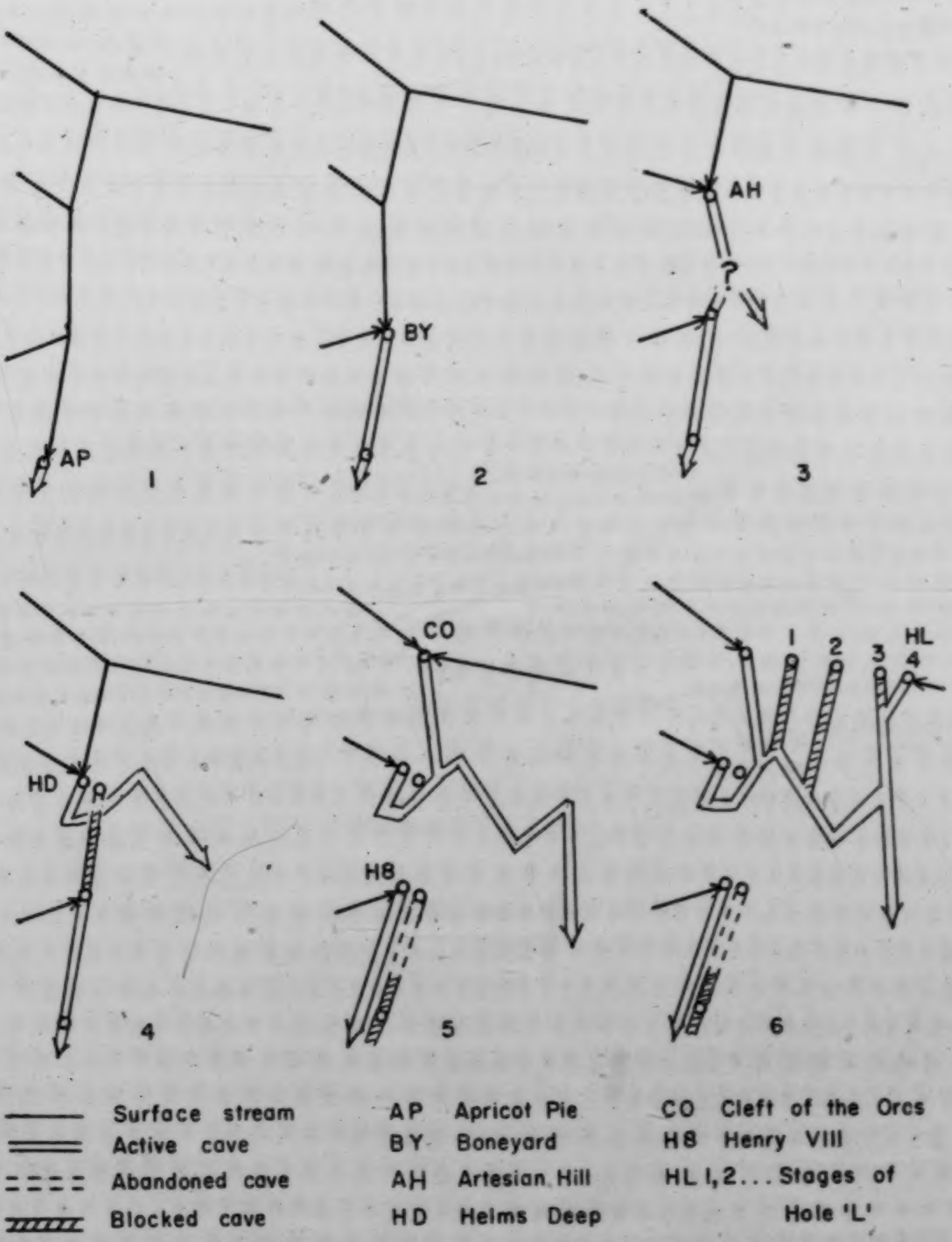

AP
1
BY
2
AH
?
3
HD
4
CO
H8
5
1
2
3
HL
4
6
Surface stream
Active cave
Abandoned cave
Blocked cave
AP Apricot Pie
BY Boneyard
AH Artesian Hill
HD Helms Deep
CO Cleft of the Orcs
H8 Henry VIII
HL 1, 2... Stages of Hole 'L'

FIGURE 10. SPECULATIVE HISTORY OF THE DEVELOPMENT OF CATCHMENT OF UPPER GARDNERS GUT

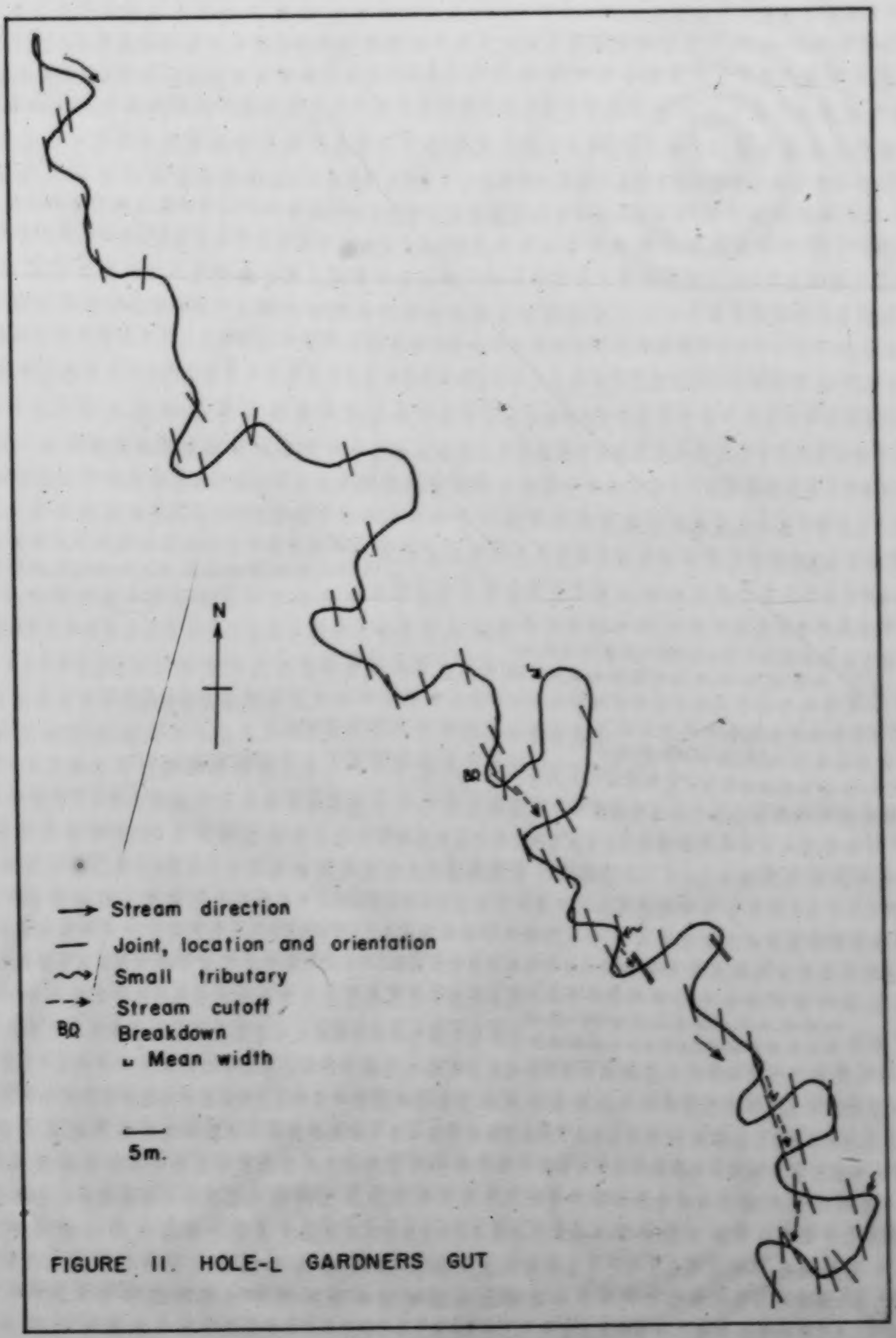

N
Stream direction
Joint, location and orientation
Small tributary
Stream cutoff
BD Breakdown
Mean width
5m.
BD

FIGURE 11. HOLE-L GARDNERS GUT

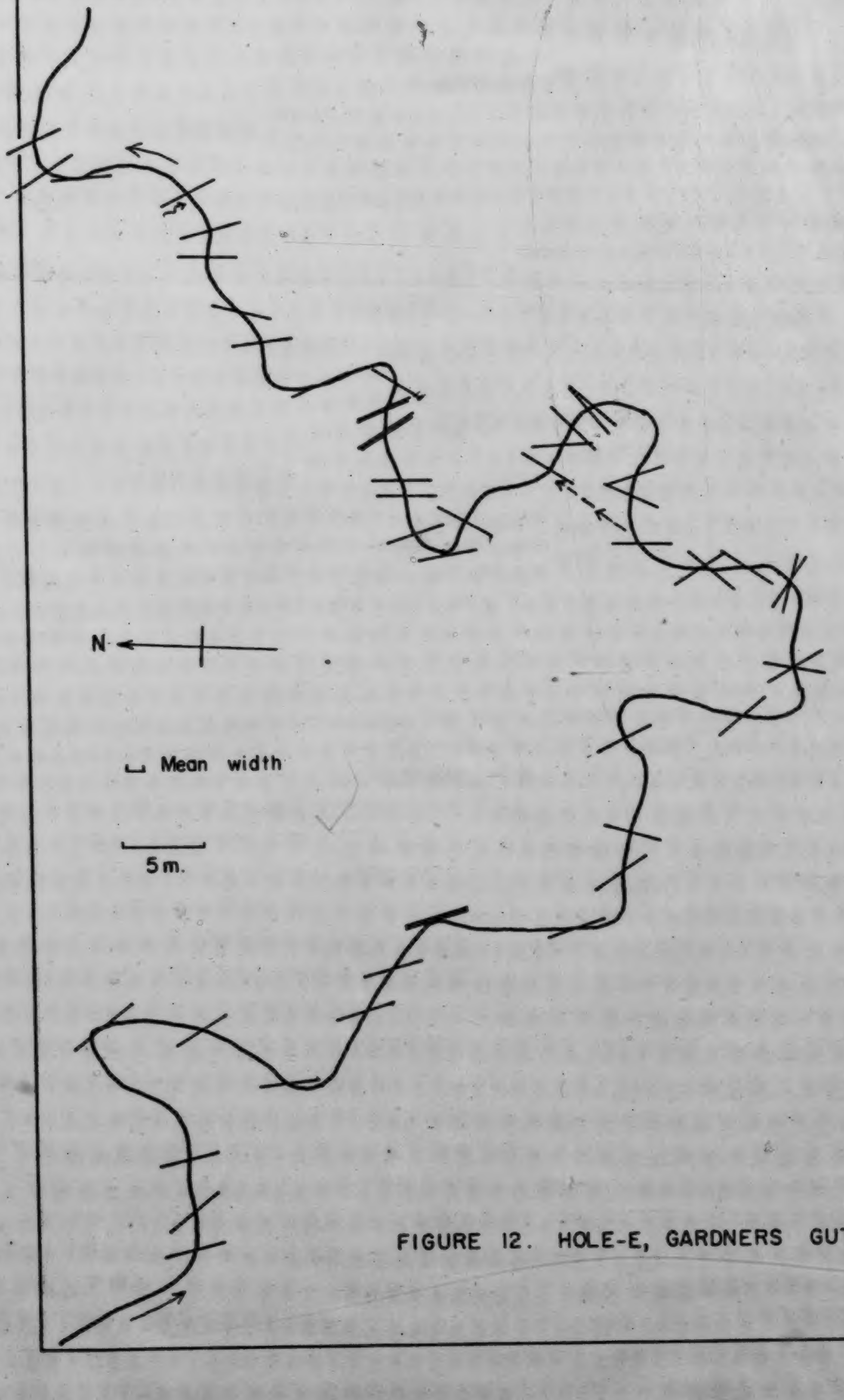

N
Mean width
5 m.

FIGURE 12. HOLE-E, GARDNERS GUT

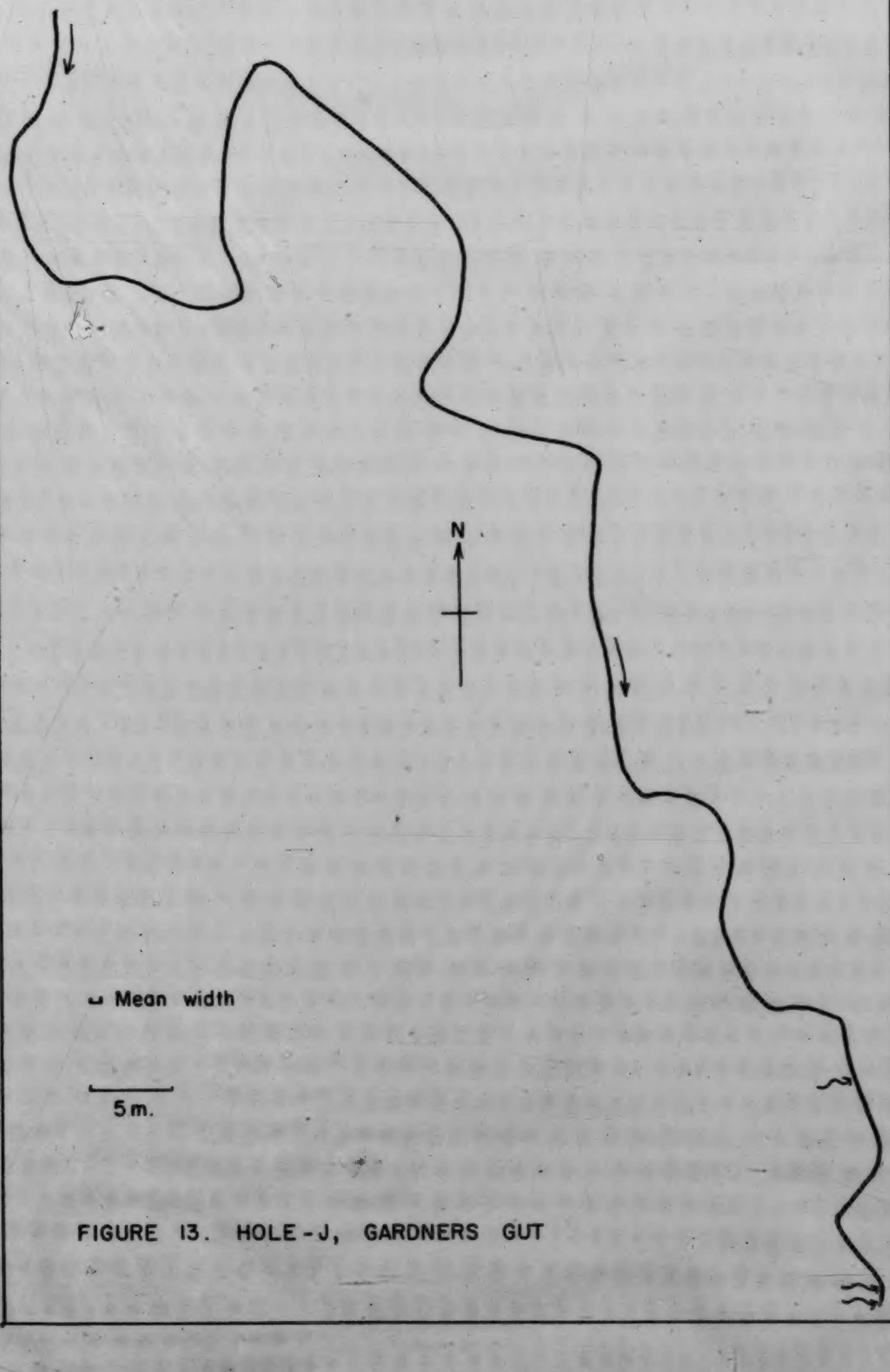

N
Mean width
5 m.

FIGURE 13. HOLE-J, GARDNERS GUT

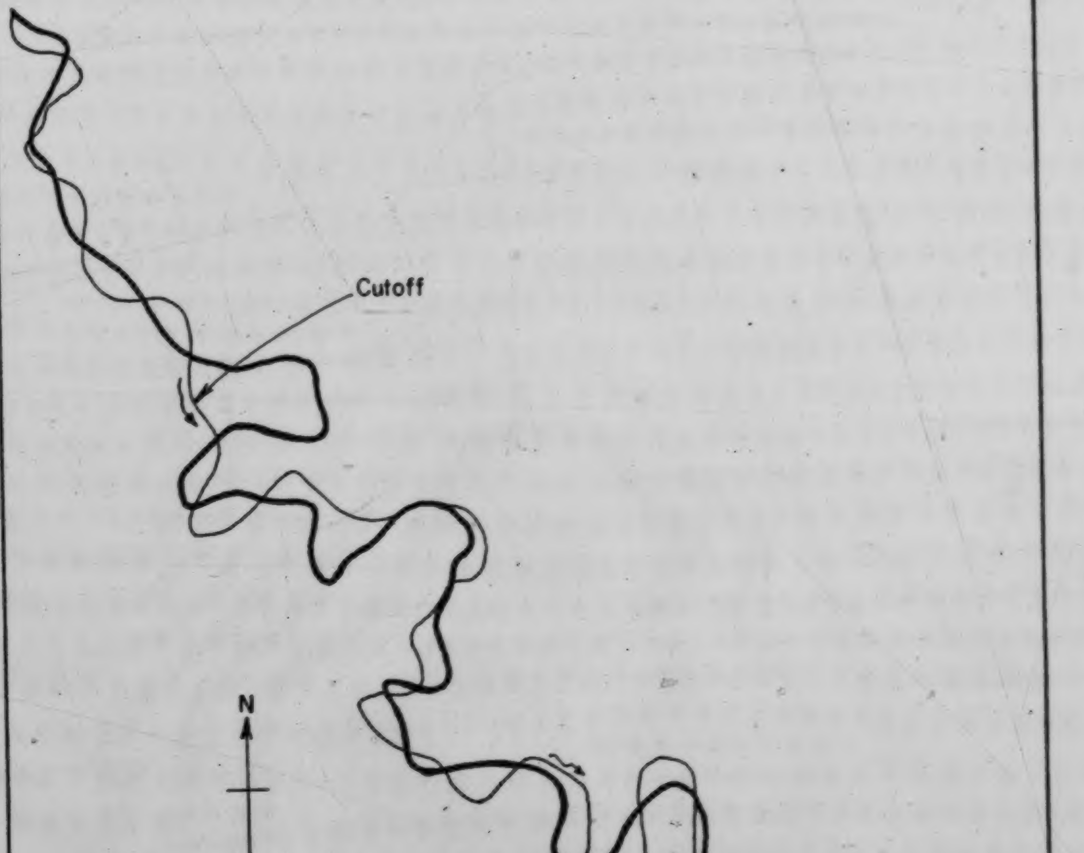

Cutoff
N
Centre line ~ 2m. above stream (Hole-N)
Present centreline (Hole-L)
Mean width
5m.

FIGURE 14. MIGRATION OF HOLE-L FROM HOLE-N, GARDNERS GUT

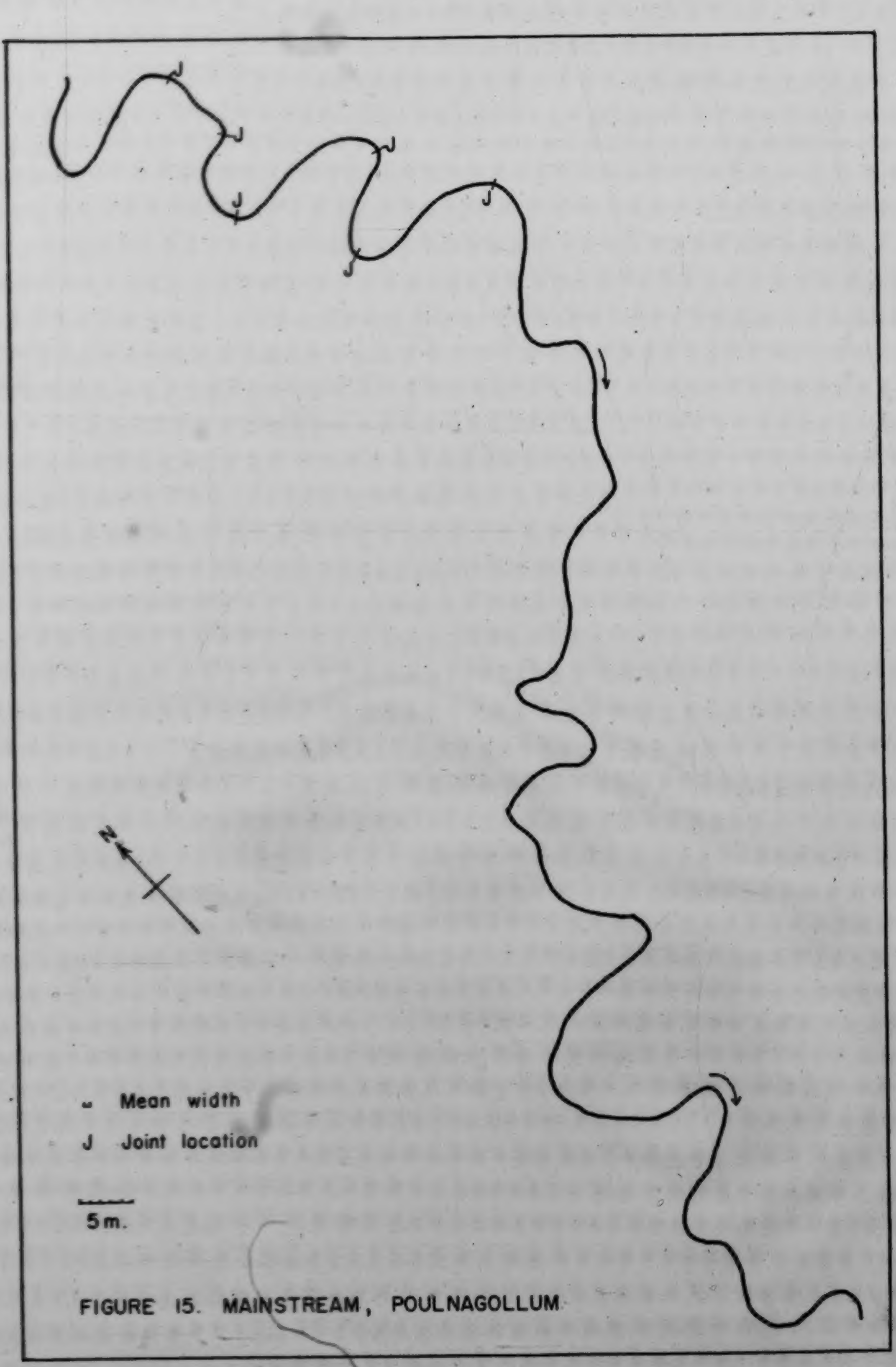

J
J
J
J
J
J
N
Mean width
J Joint location
5m.

FIGURE 15. MAINSTREAM, POULNAGOLLUM

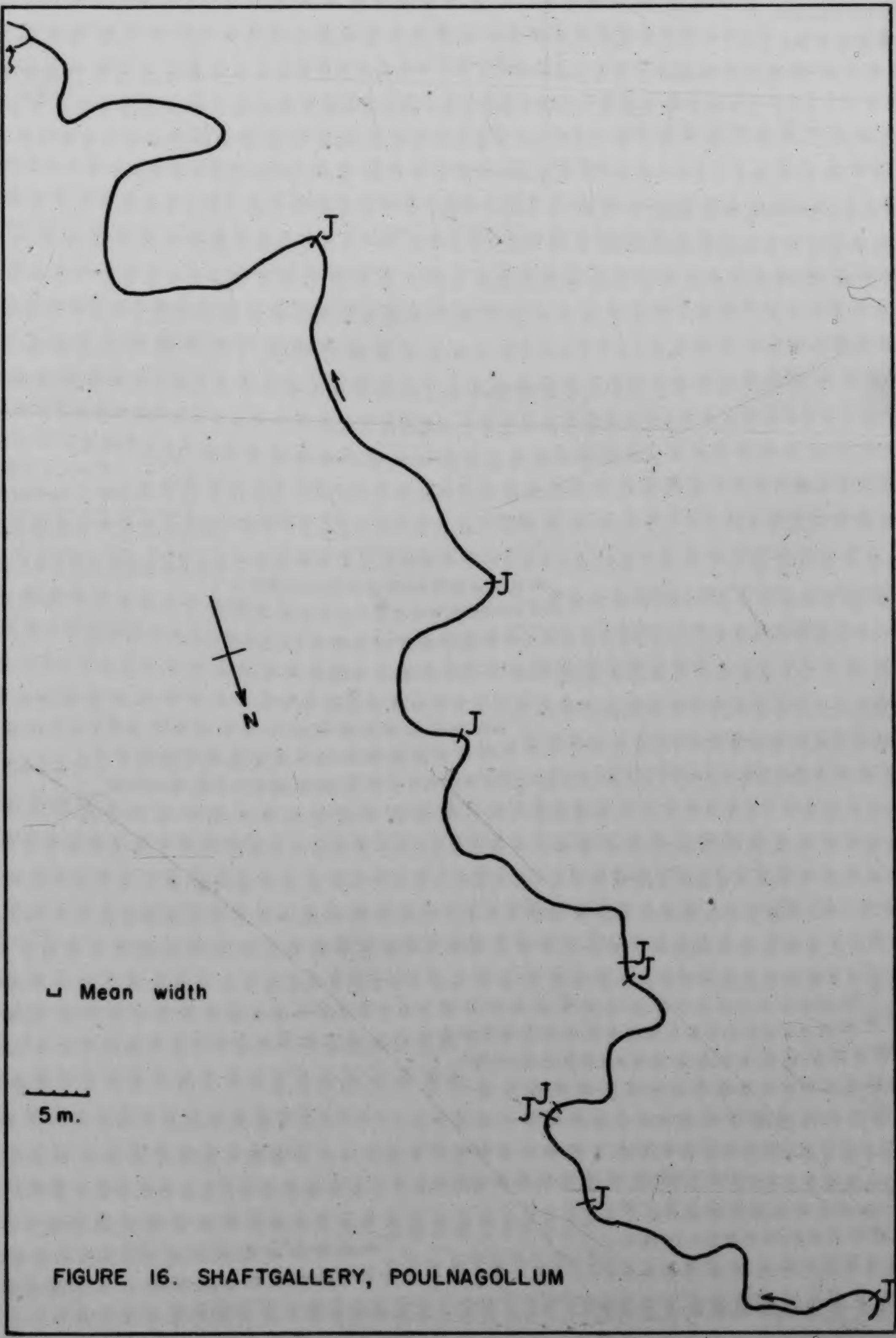

J
J
J
J
J
J
J
J
J
N
⊔ Mean width
5 m.

FIGURE 16. SHAFTGALLERY, POULNAGOLLUM

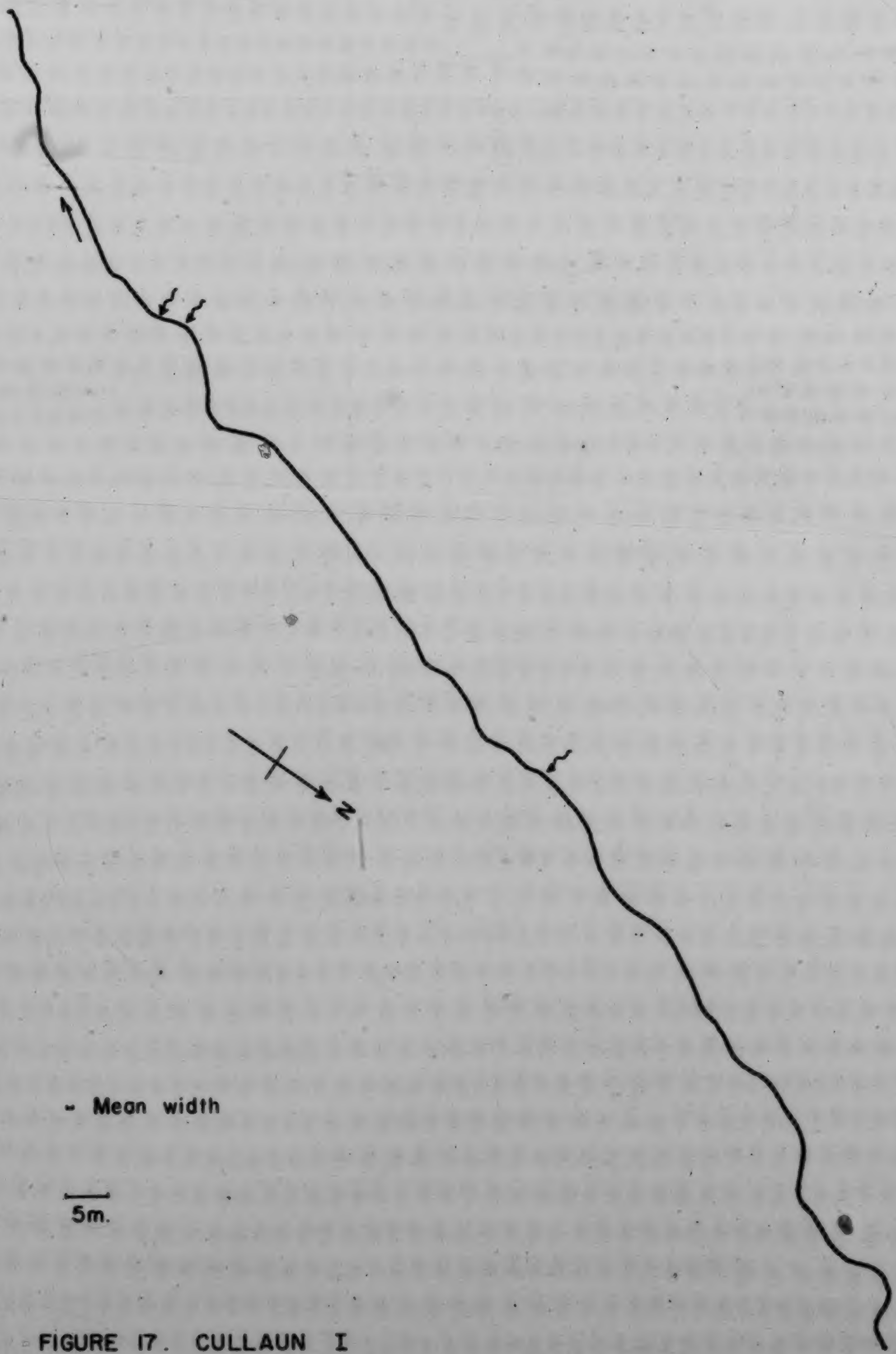

N
- Mean width
5m.

FIGURE 17. CULLAUN I

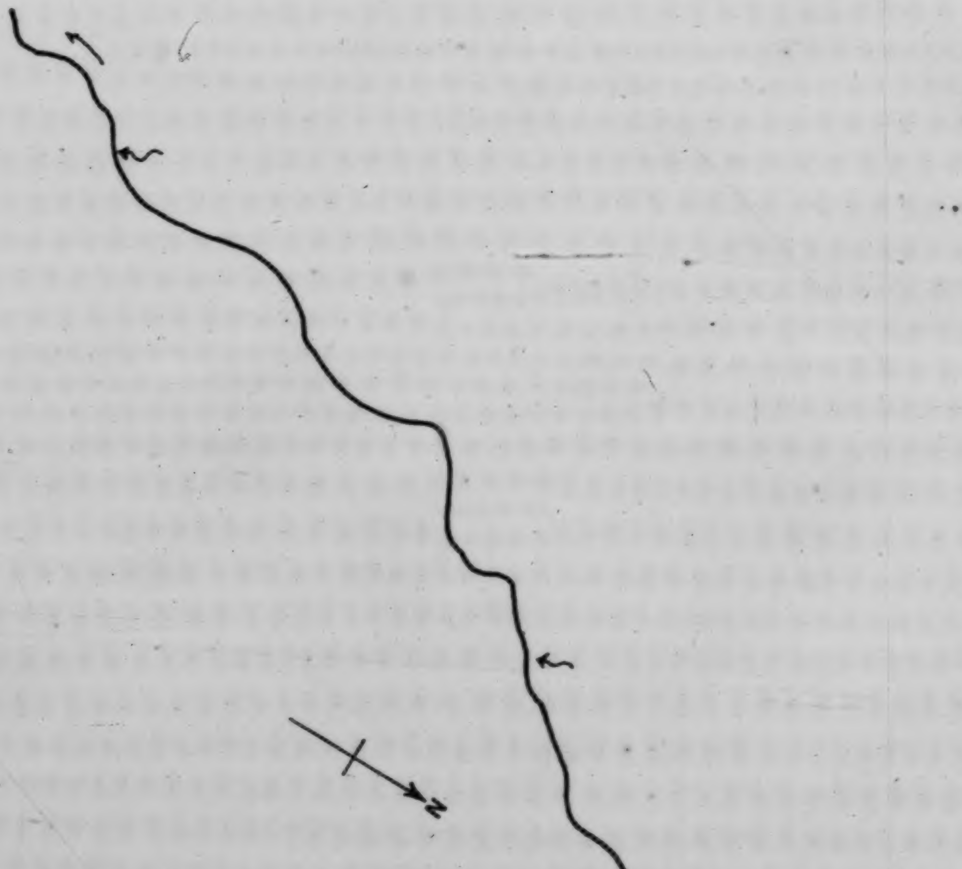

N
- Mean width
5m

FIGURE 18. ST. CATHERINES

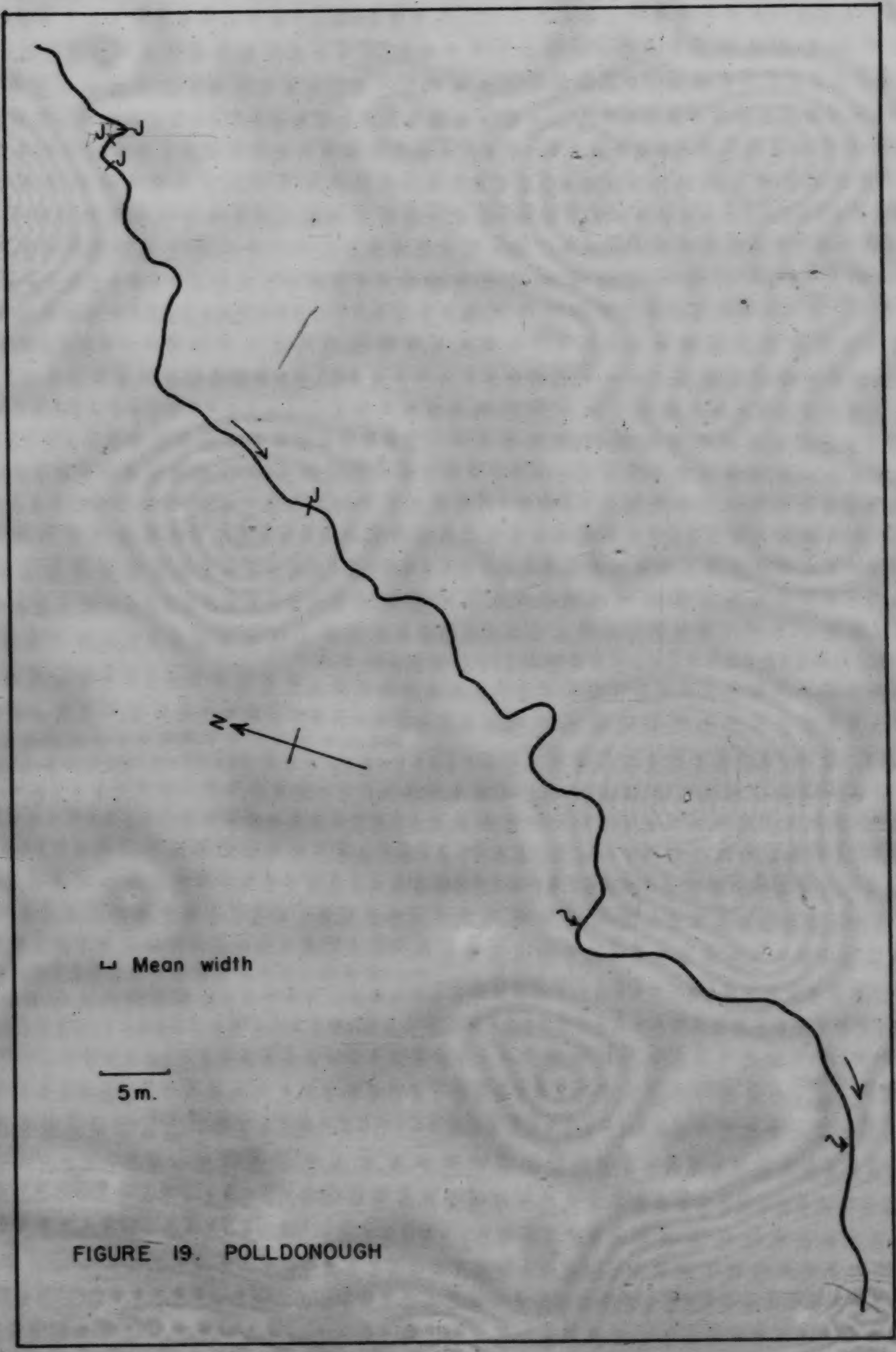

J
J
J
J
N
Mean width
5 m.

FIGURE 19. POLLDONOUGH

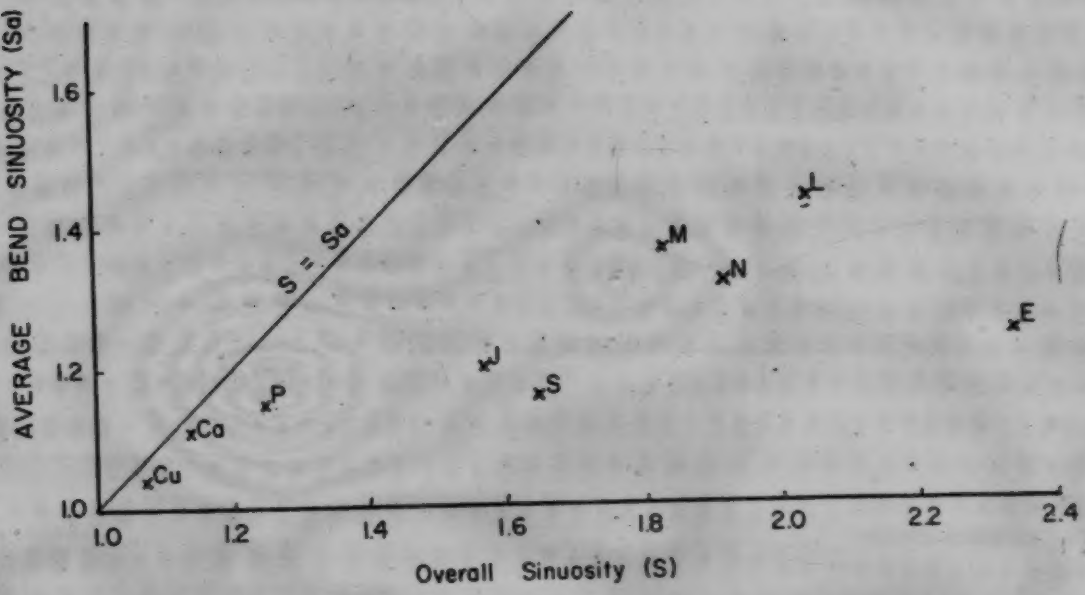


FIGURE 20a. AVERAGE BEND SINUOSITY AND OVERALL SINUOSITY

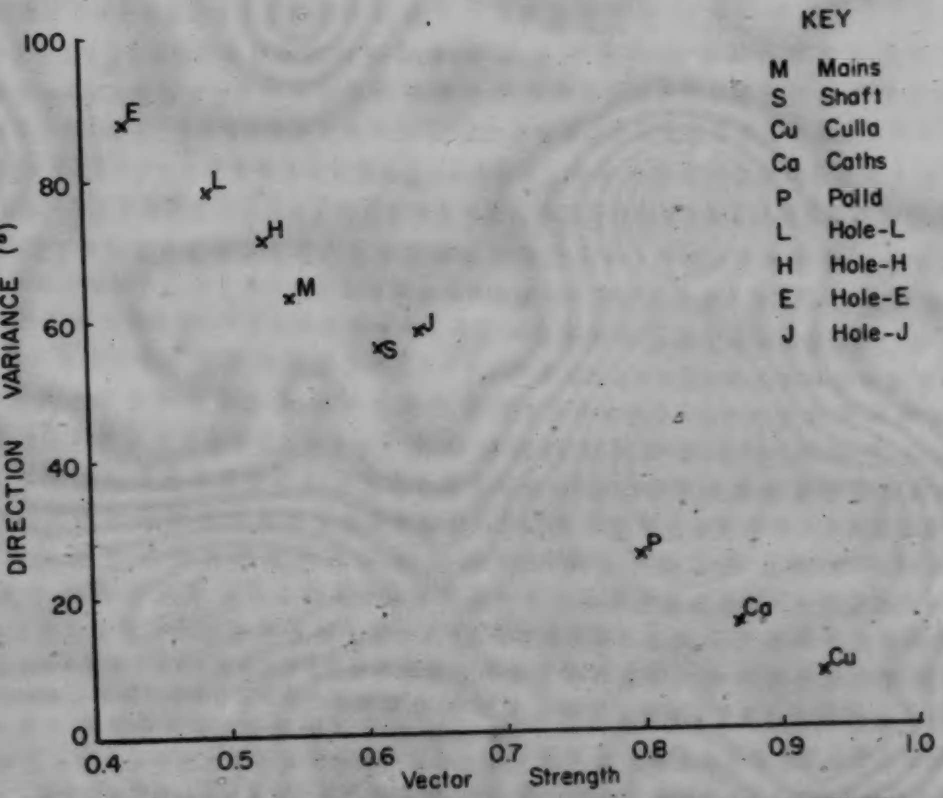
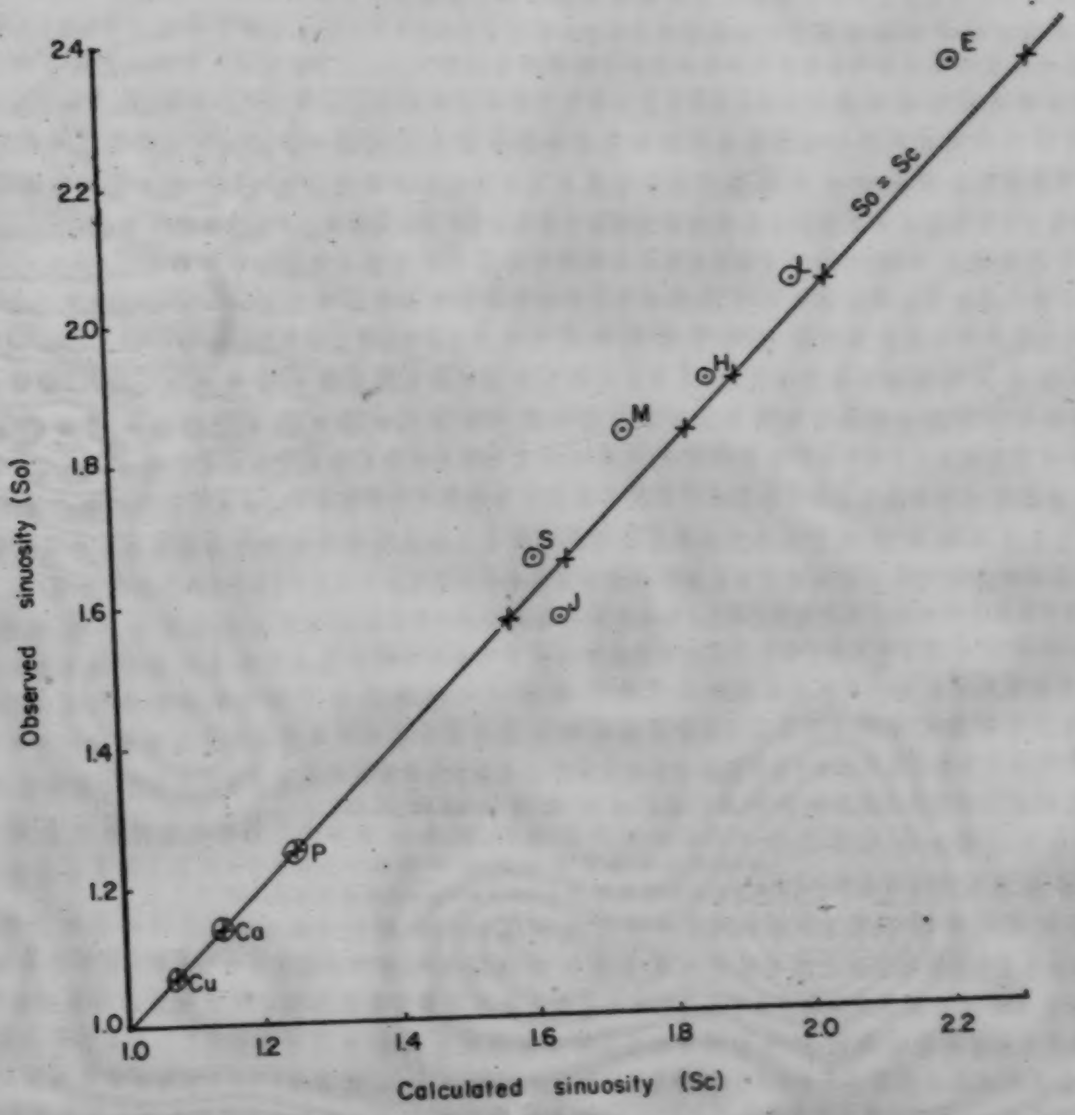


FIGURE 20b. DIRECTION VARIANCE AND VECTOR STRENGTH



KEY: ⊙ Sinuosity calculated from direction variance

+ Sinuosity calculated from vector strength

FIGURE 21. CALCULATED SINUOSITY AND OBSERVED SINUOSITY

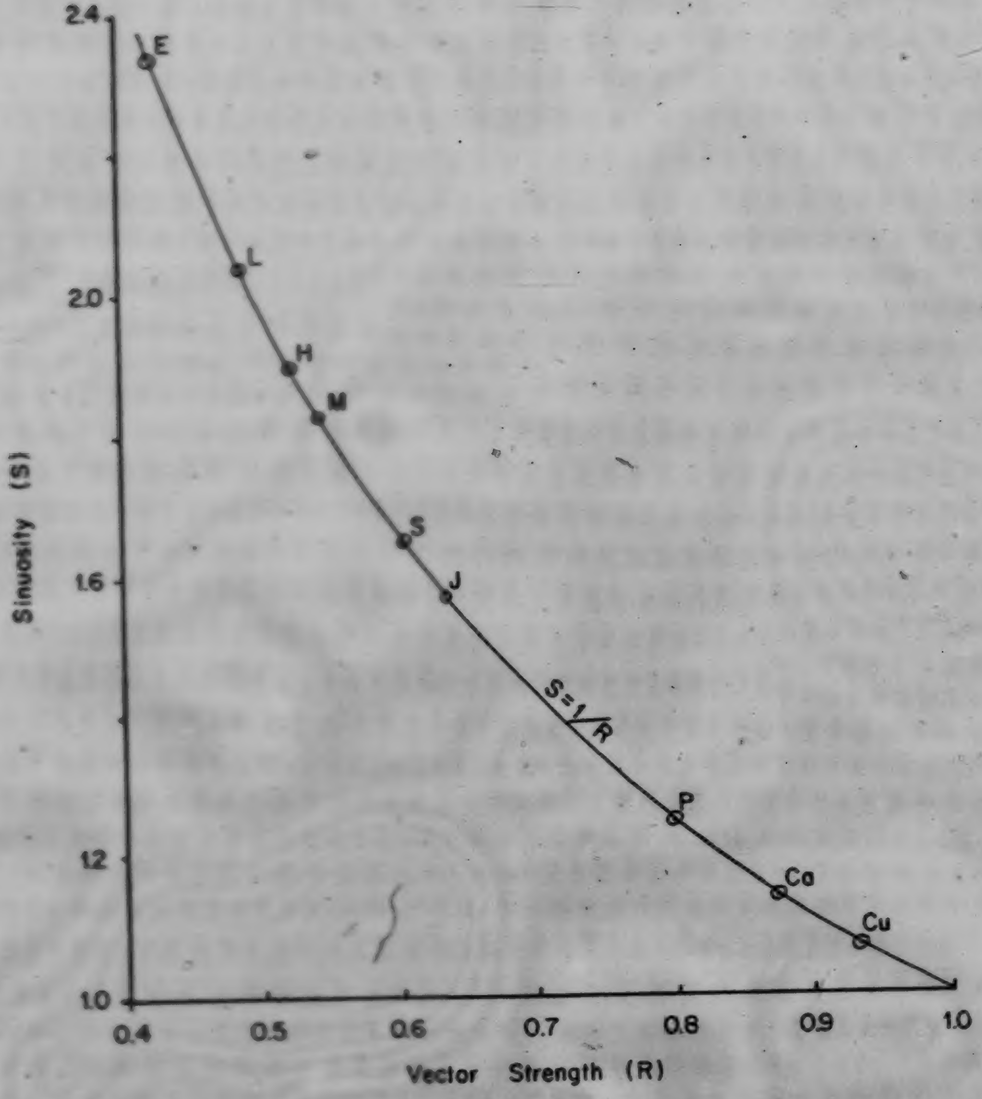


FIGURE 22. VECTOR STRENGTH AND SINUOSITY

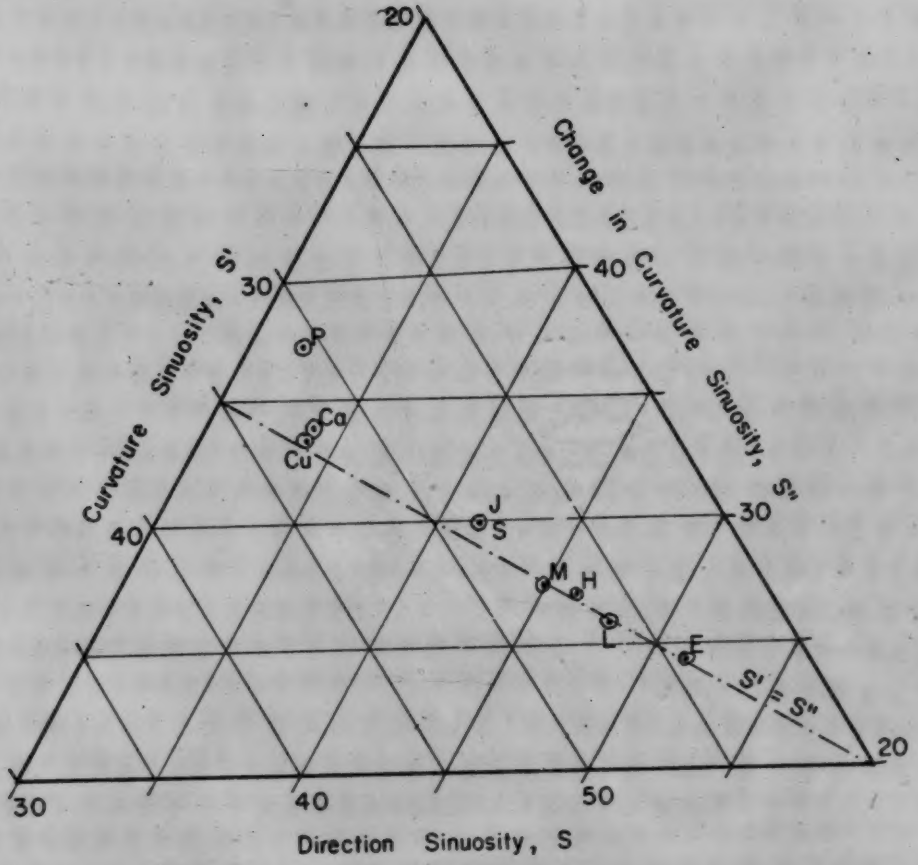


FIGURE 23a. TRIAXIAL SINUOSITY DIAGRAM

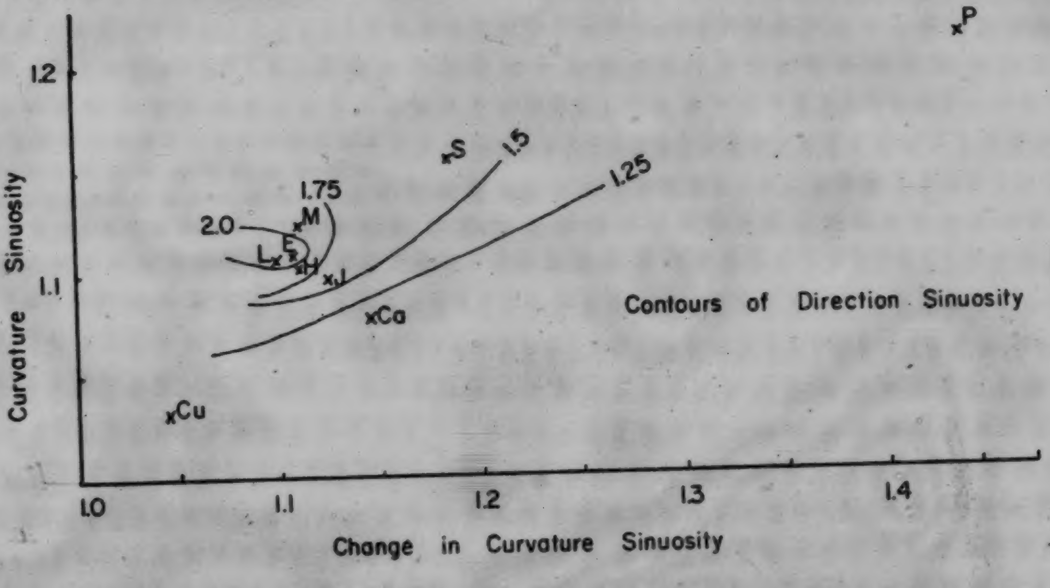


FIGURE 23b DIRECTION, CURVATURE AND CHANGE IN CURVATURE SINUOSITY

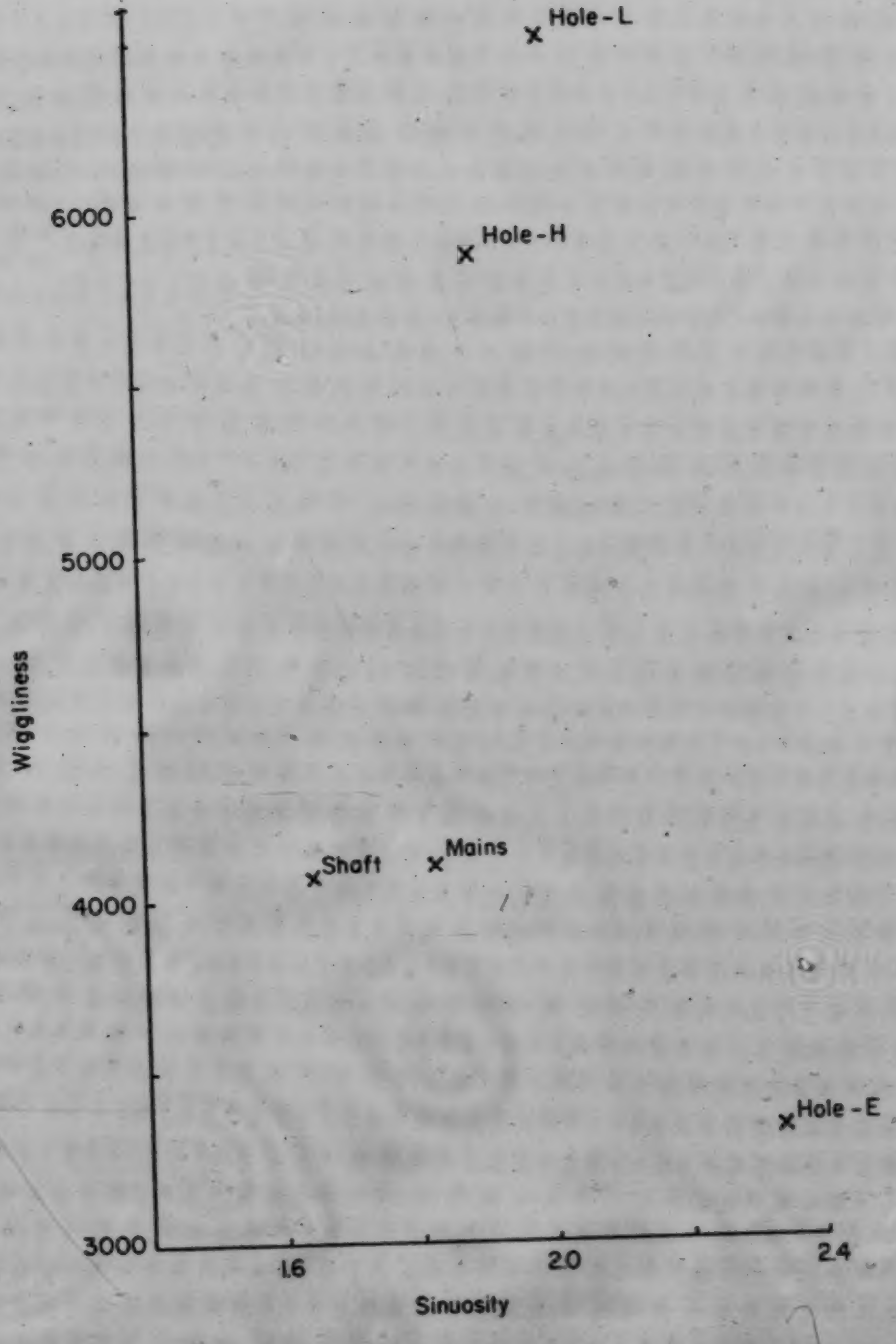


FIGURE 24. SINUOSITY AND WIGGLINESS

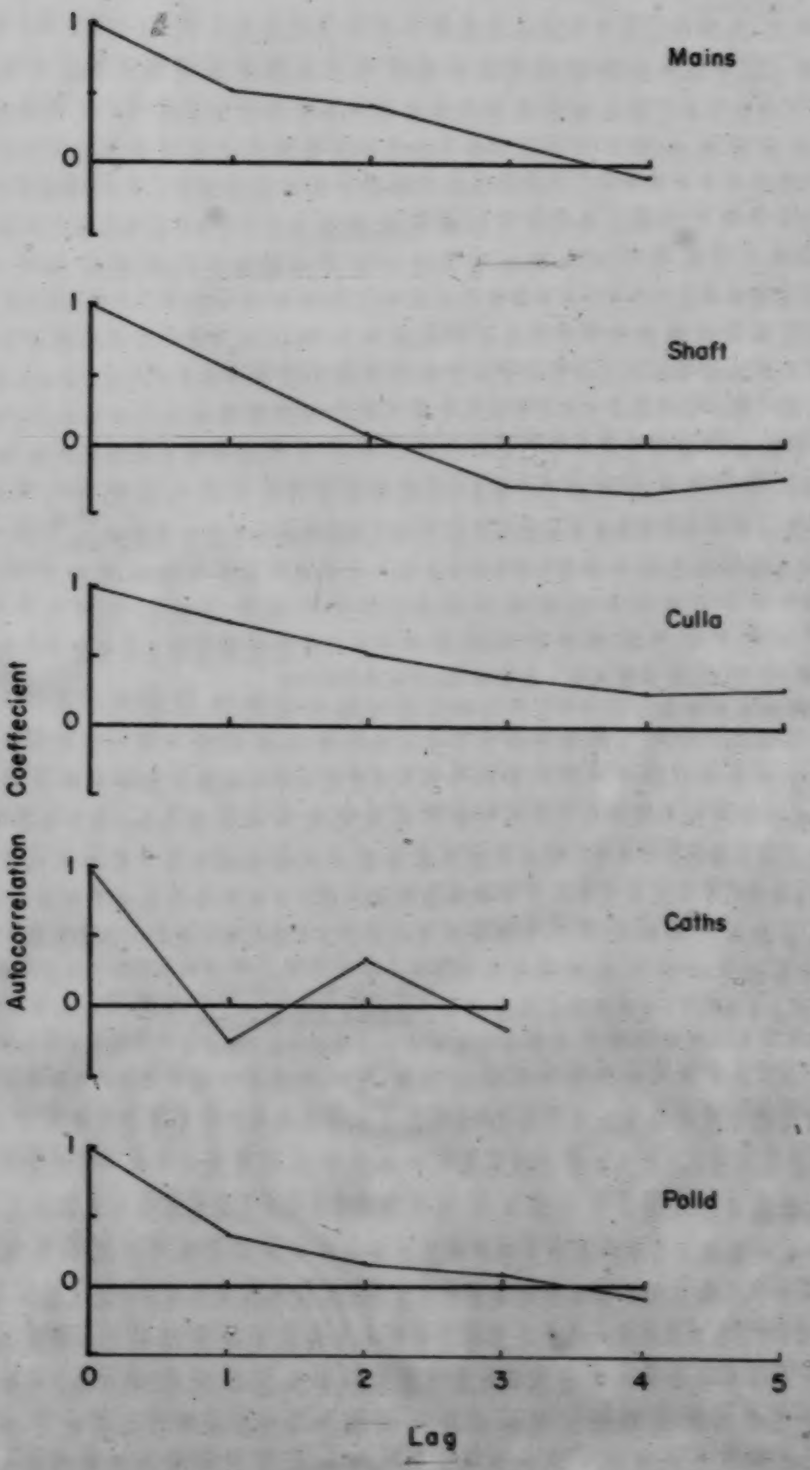


FIGURE 25. AUTOCORRELATION OF WIDTH FOR INDIVIDUAL BENDS

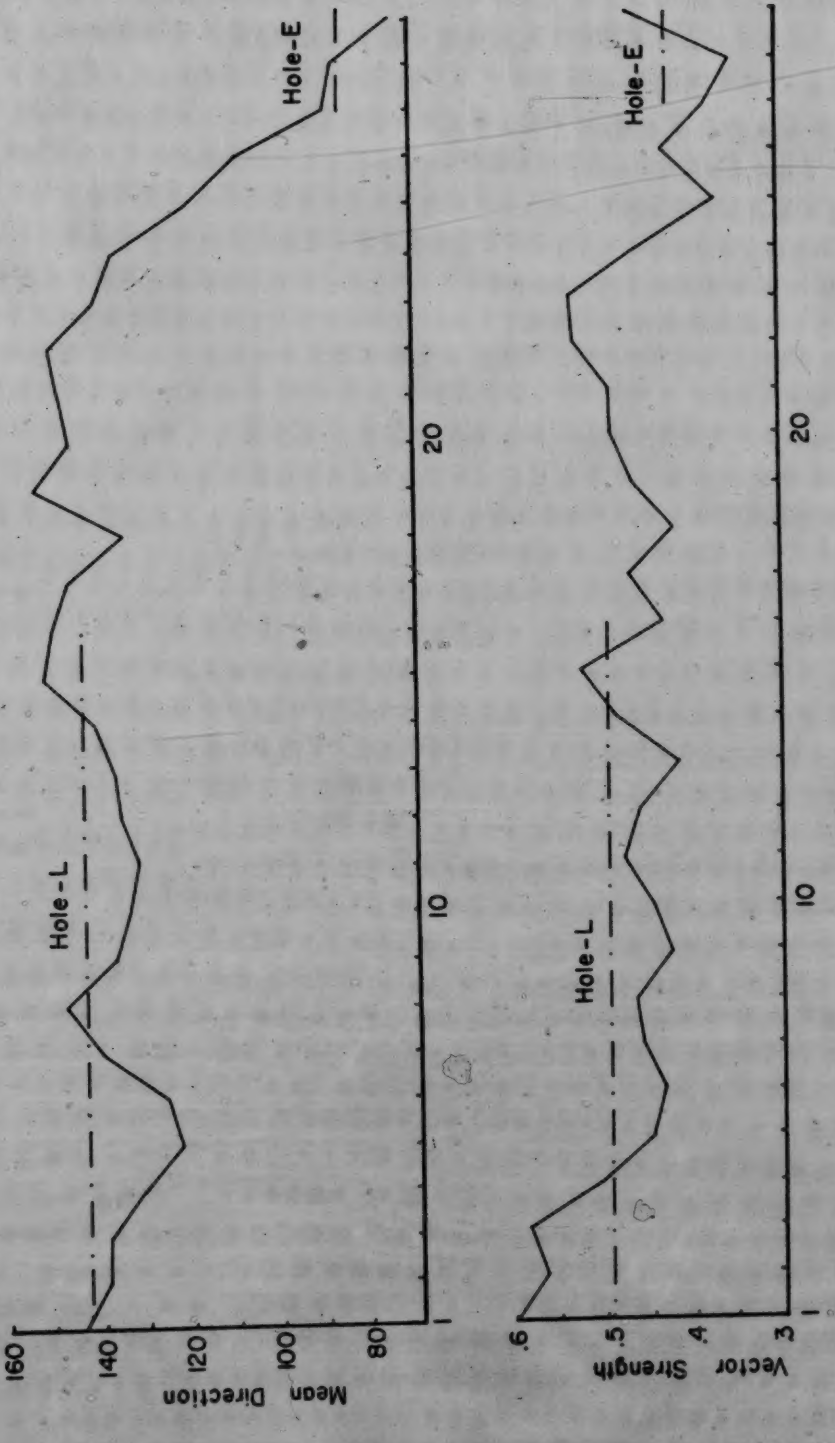
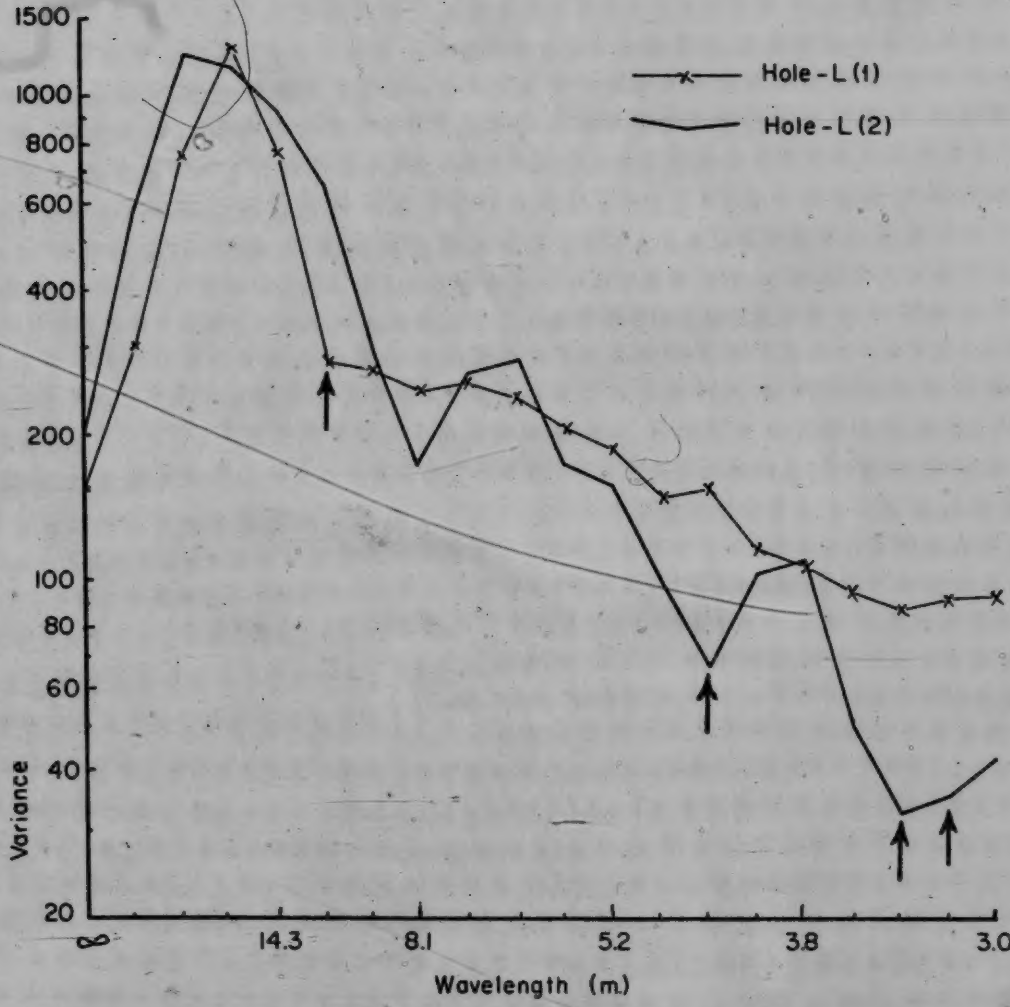


FIGURE 26. RUNNING MEAN AND VECTOR STRENGTH:
HOLE-L AND HOLE-E



↑ signifies estimates significantly different at .05

FIGURE 27. SPECTRA OF TWO HALVES OF HOLE - L

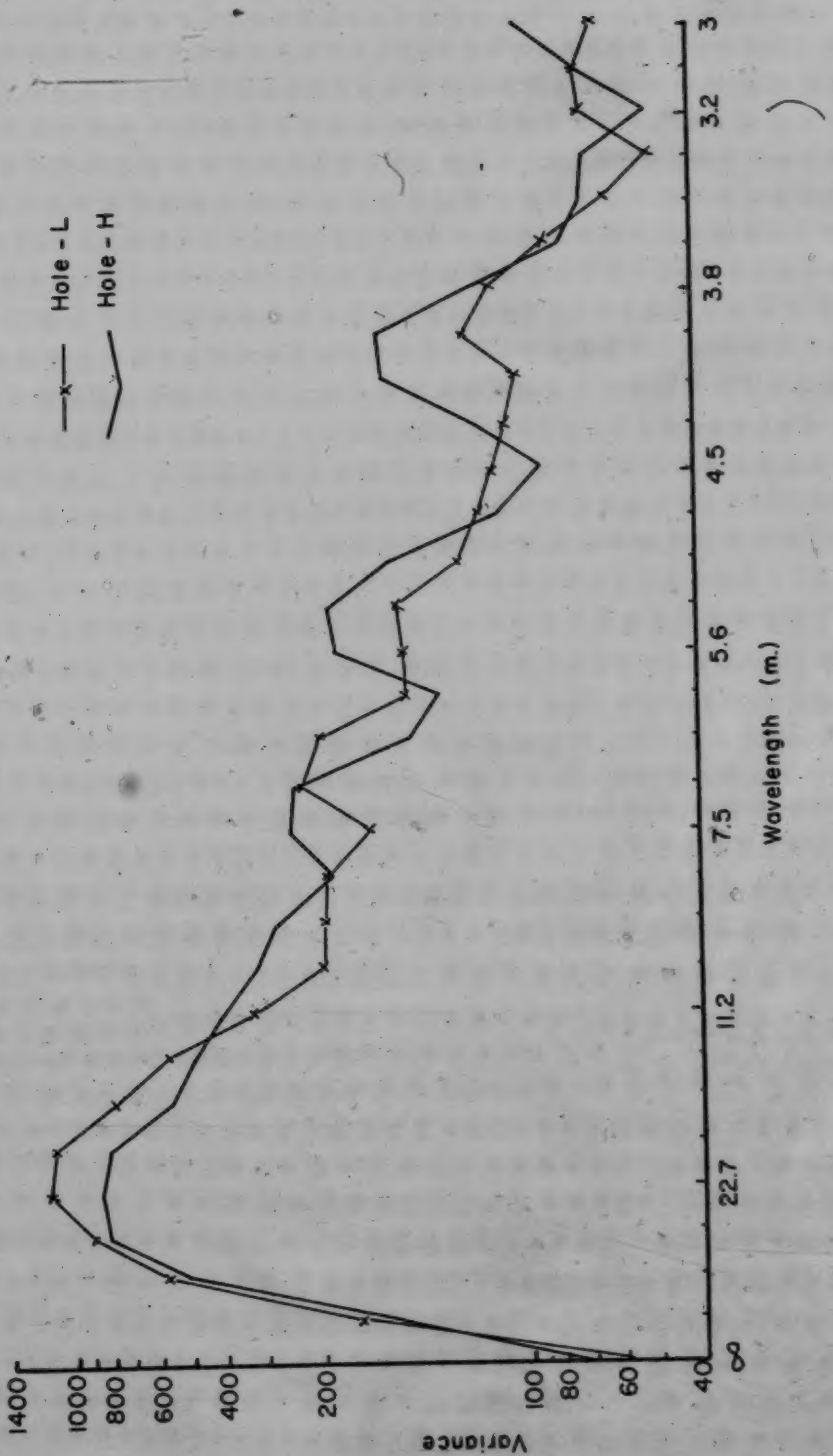
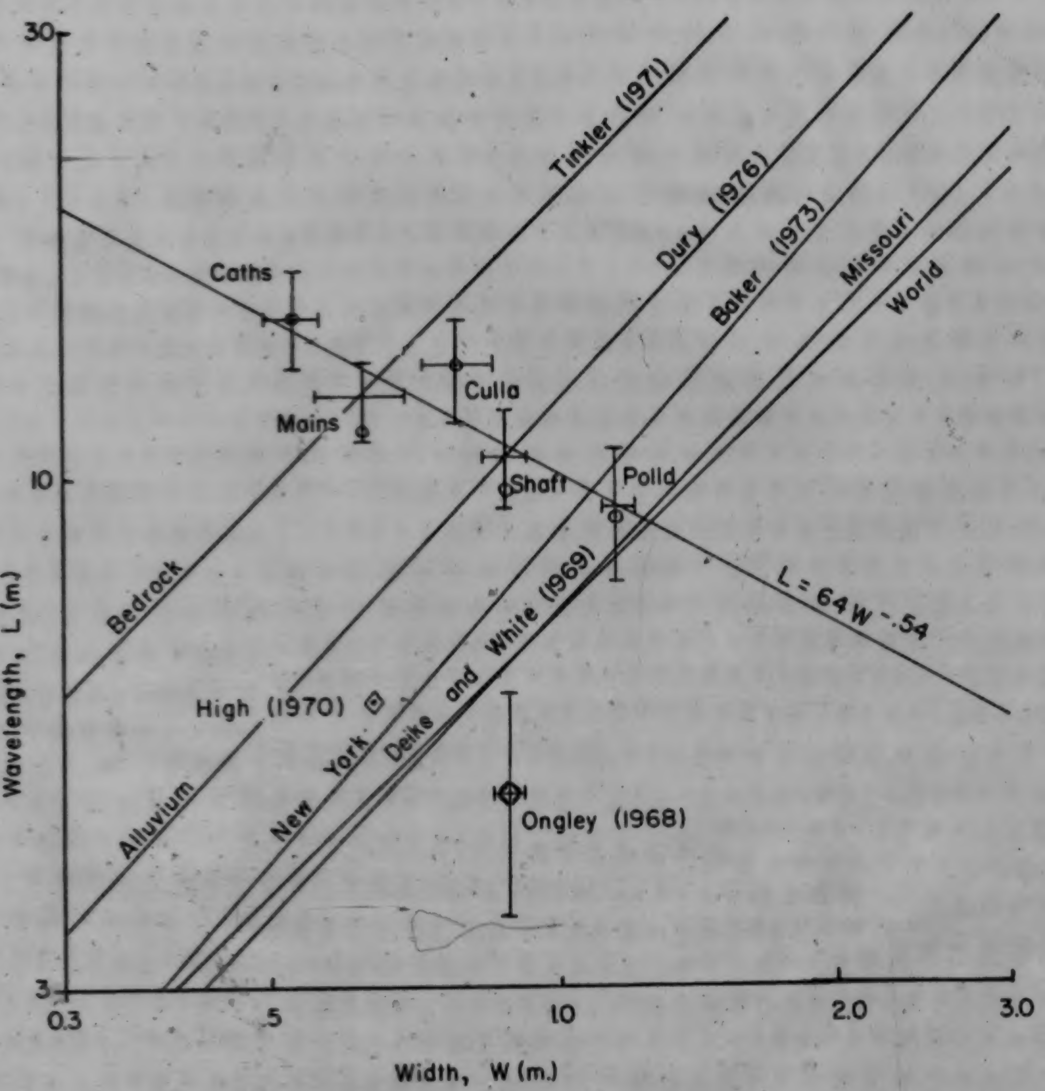


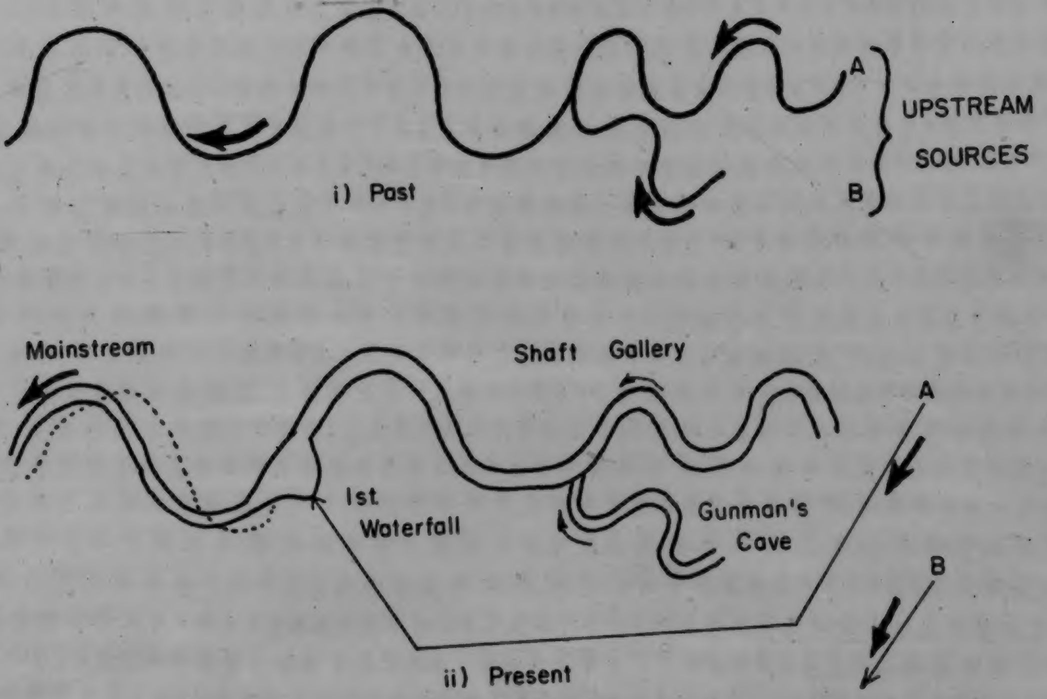
FIGURE 28. SPECTRA OF HOLE - L AND HOLE - H



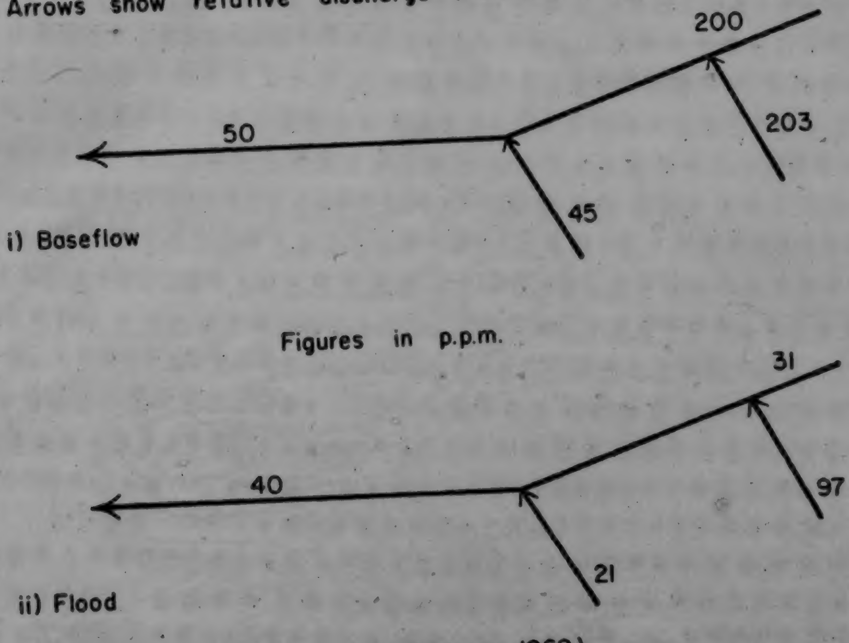
Bars denote one standard deviation

Circles are wavelengths over 2 consecutive bends

FIGURE 29. WIDTH AND WAVELENGTH RELATIONSHIPS FROM THE LITERATURE AND PRESENT WORK


A
UPSTREAM SOURCES
B
i) Past
Mainstream
Shaft Gallery
A
1st Waterfall
Gunman's Cave
B
ii) Present

a) Arrows show relative discharge (Palaeohydrology)


200
50
203
45
i) Baseflow
Figures in p.p.m.
31
40
97
21
ii) Flood

b) Summer Hydrochemistry (by Tratman 1969)

FIGURE 30. PALAEOHYDROLOGY AND SUMMER HYDROCHEMISTRY OF SHAFTGALLERY AND MAINSTREAM (POULNAGOLLUM)

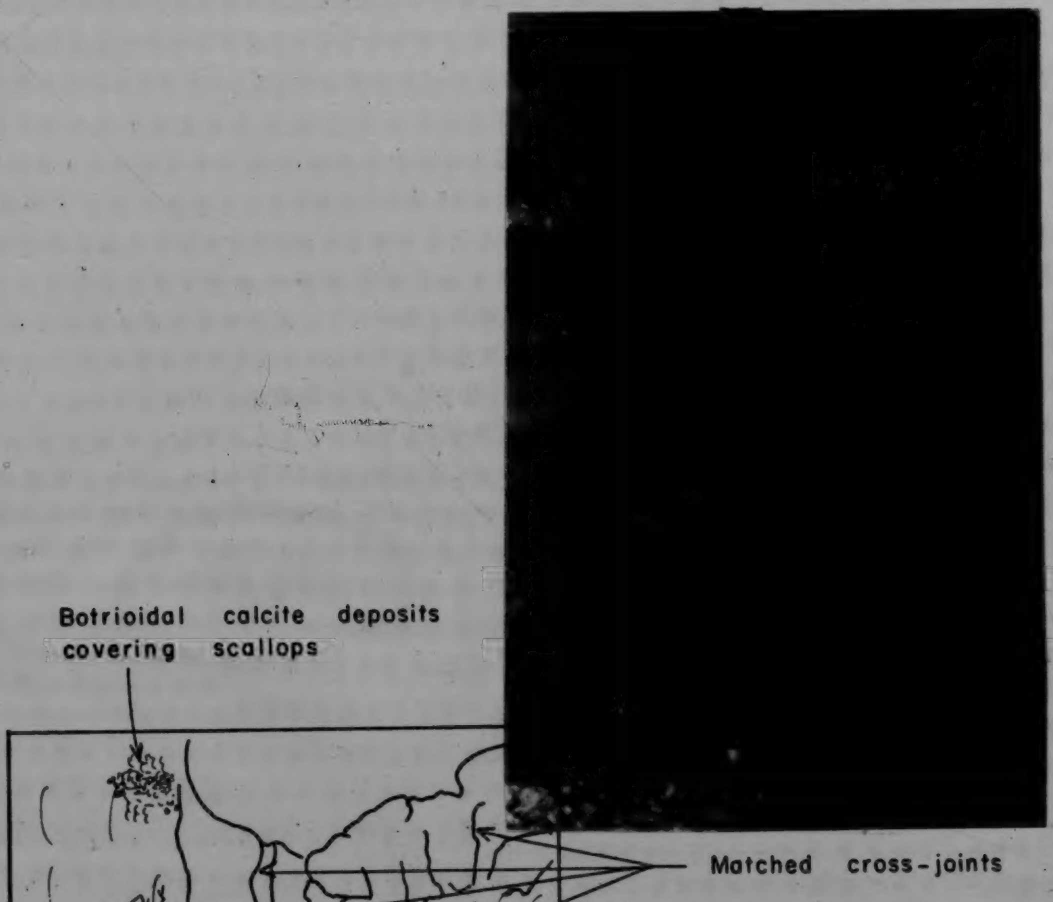

Botrioidal calcite deposits covering scallops
Matched cross-joints
Ancient bifurcation
Flutes
30 cm.
Abundant scalloping, note rock type
Stream meander

PLATE I. SCALLOPS AND FLUTES IN A SMALL CAVE MEANDER

PLATE 2. A SMALL INLET ABOVE THE STREAM. NOTE PROMINENCE OF RESIDUAL BEDS. (GARDNERS GUT)

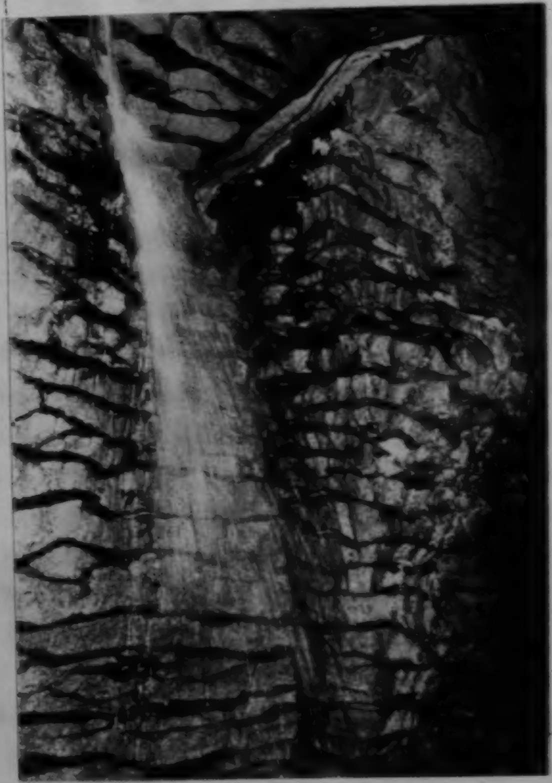


PLATE 3. SURFACE OUTCROP OF OTOROHANGA LIMESTONE. NOTE PREFERENTIAL EROSION OF RESIDUAL BEDS.

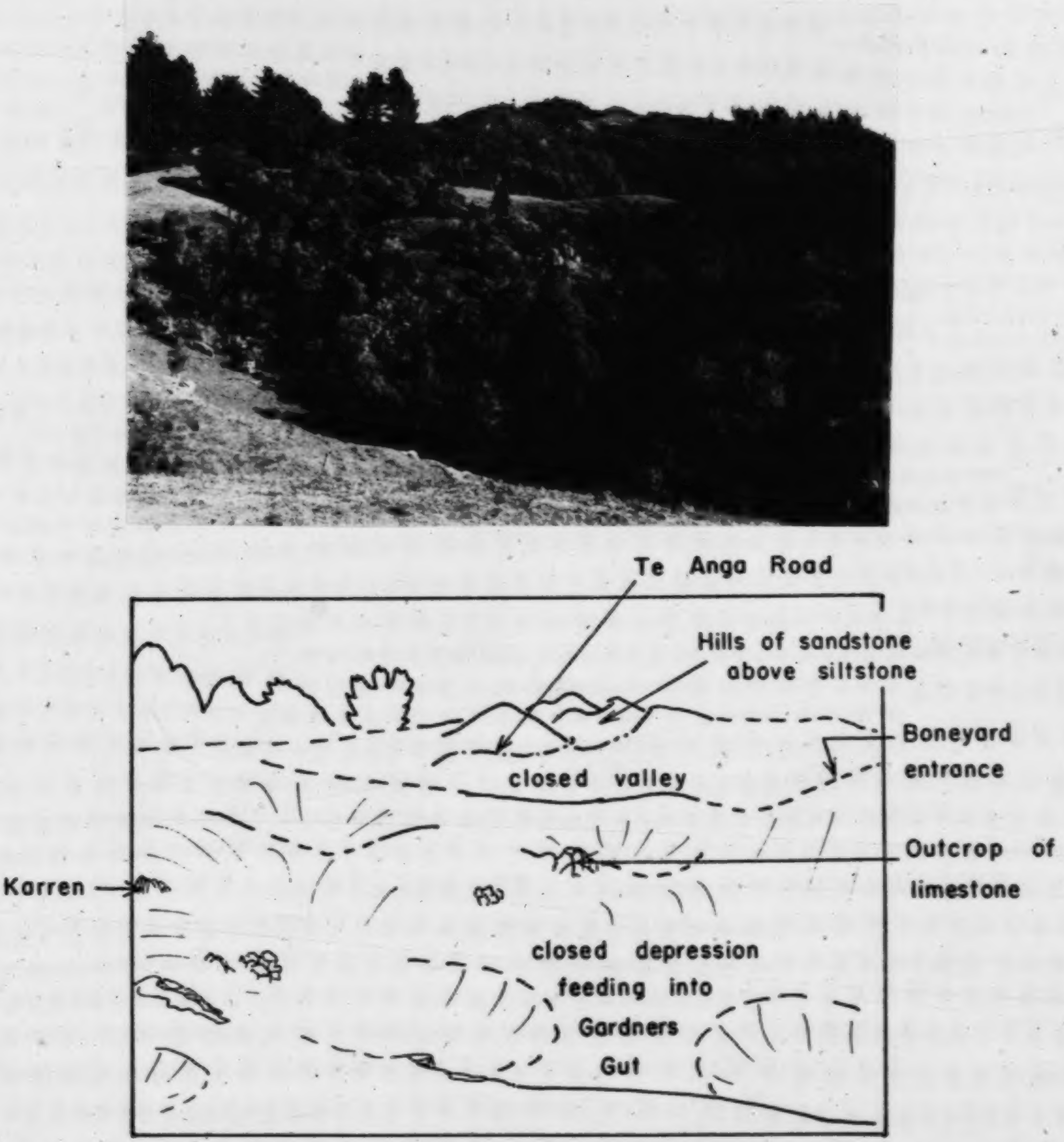

Te Anga Road
Hills of sandstone above siltstone
Boneyard entrance
closed valley
Outcrop of limestone
Karren
closed depression feeding into Gardners Gut

PLATE 4. SURFACE GEOMORPHOLOGY ABOVE GARDNERS GUT. PHOTOGRAPH TAKEN LOOKING WEST FROM NEAR EXIT SEVEN.

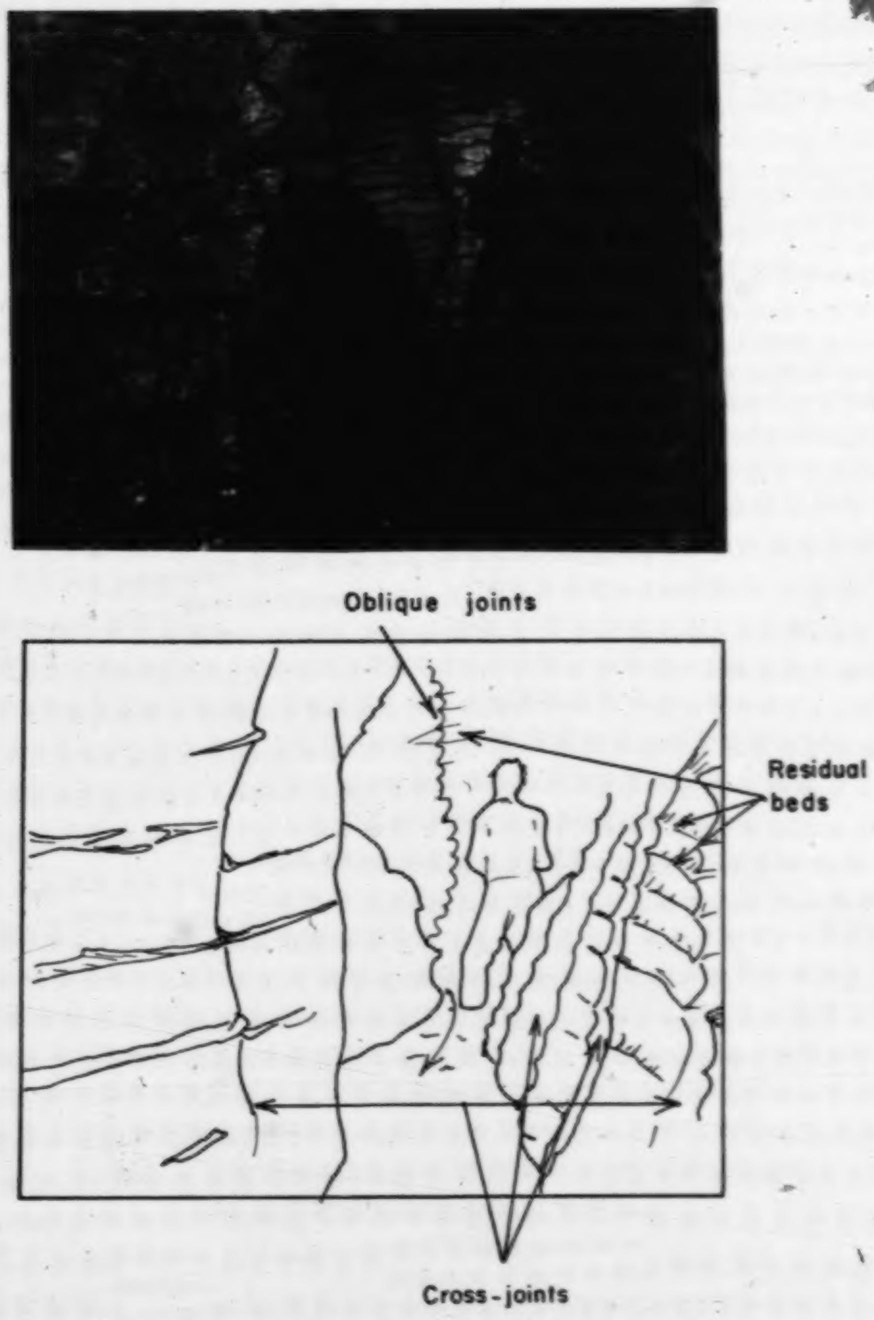

Oblique joints
Residual beds
Cross-joints

PLATE 5. STREAM IN AREA OF JOINT CONTROL
(SURVEYED AS HOLE-J)

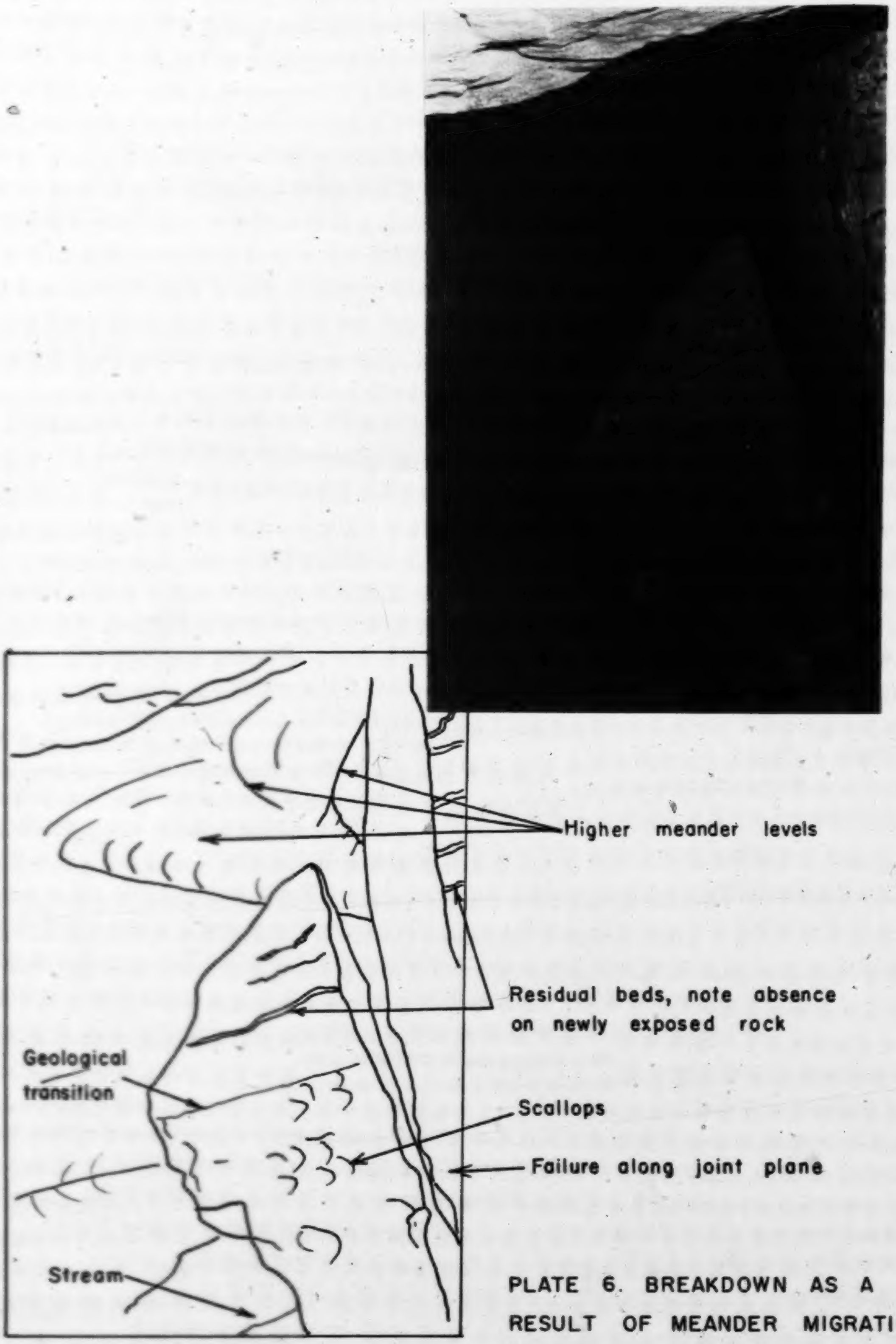

Higher meander levels
Residual beds, note absence on newly exposed rock
Geological transition
Scallops
Failure along joint plane
Stream

PLATE 6. BREAKDOWN AS A RESULT OF MEANDER MIGRATION

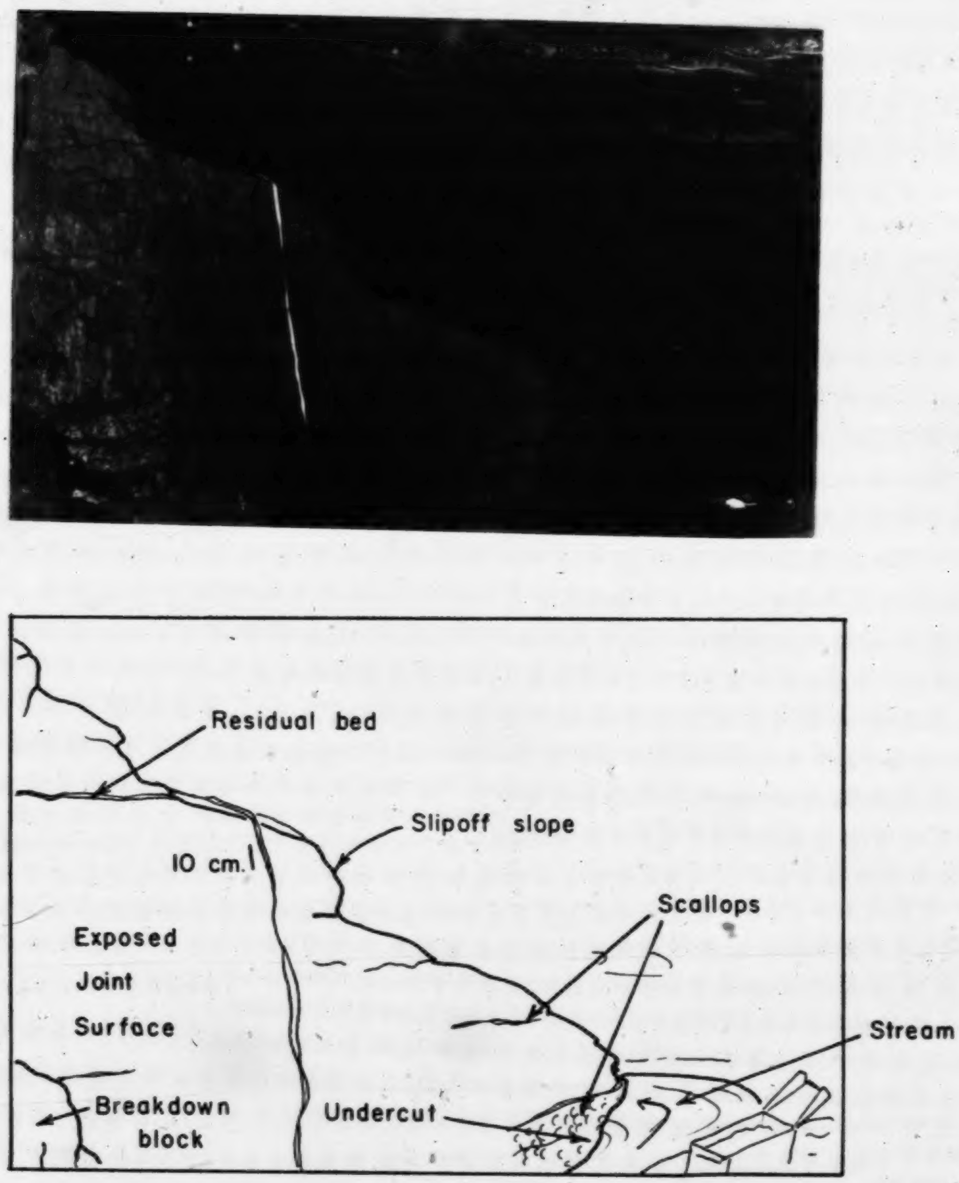

Residual bed
Slipoff slope
10 cm.
Scallops
Exposed
Joint
Surface
Stream
Breakdown
block
Undercut

PLATE 7. JOINT SURFACE EXPOSED BY BREAKDOWN-BLOCK OF PLATE 6.

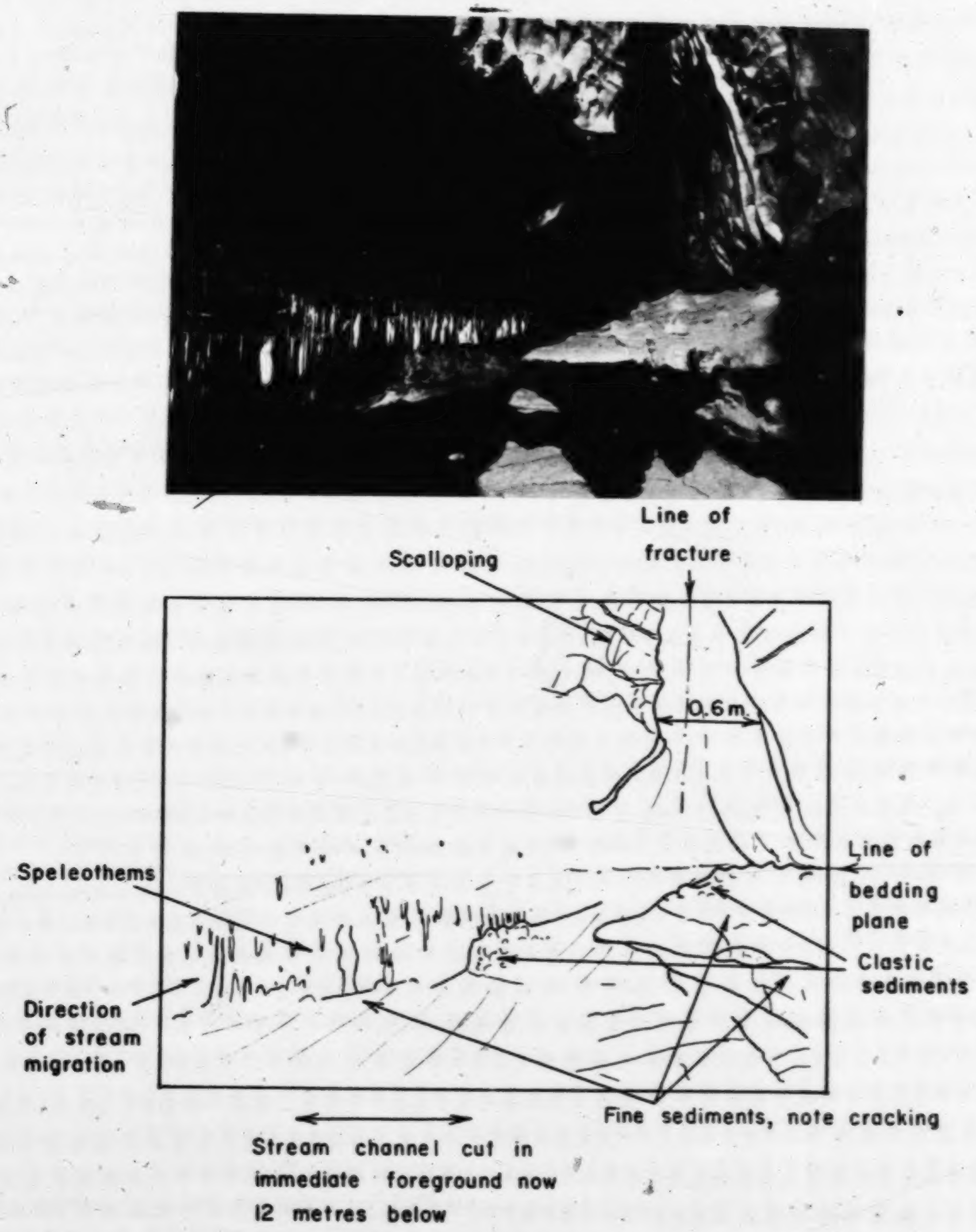

Line of fracture
Scalloping
0.6 m
Line of bedding plane
Speleothems
Clastic sediments
Direction of stream migration
Fine sediments, note cracking
Stream channel cut in immediate foreground now 12 metres below

PLATE 8. ROOF OF PART OF GARDNERS GUT SHOWING FRACTURE FROM WHICH CAVE WAS INITIATED (see Fig. 4a)

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APPENDIX A

DATA AVAILABILITY

The bearing, width and individual bend data are stored on a Geography Department Tape held by the University of Alberta Computing Services. This is an MTS tape, documented under *FS, a straightforward tape and file handling routine. The designation of this tape and the filenames under which data are stored are available from the author on request.